Generation of drifting optical pulses

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Abstract: The radiation in the form of Airy and Gaussian optical pulses is investigated. It is shown that the pulse envelope moves decelerating in the time-spatial coordinates on the surface of a parabolic cylinder for the Airy pulse and a hyperbolic cylinder for the Gaussian.

I. INTRODUCTION

Intensive theoretical and experimental investigations of Airy beams are motivated by their unusual features (non-diffractive propagation, accelerating motion, and self-healing) [1-4]. Some very interesting applications are already realised [5-10]. In the majority of these investigations time is eliminated from the paraxial equation by using the predominant time harmonic dependence $E = \tilde{F}(t,x,y,z)e^{i(\omega t - kx)}$ for a wave propagating along the x axis. In this case a parabolic dependence exists between the lateral and the longitudinal coordinates. The parabolic dependence between time and the longitudinal coordinate with the leading dependence $\sim e^{i\omega t}$ is considered also in a number of publications [2, 5, 9, 10]. Here we suggest a different approach, considering a simple statement of the problem when electromagnetic pulses in nondispersive medium are generated by an external source. Our results provide a physically natural picture of the phenomenon and avoid the problem of backward flow of time. We do not make any assumptions on the temporal dependence of the field and can construct different decelerating pulses (not Airy only), the Gaussian is given as an example.

We consider an electromagnetic field created by an extrinsic source in a dissipative nondispersive medium with the permittivity ε, the permeability μ, and the conductivity σ. We formulate the problem in open space without initial conditions but with the source current in the plane $x = x_0$ perpendicular to the direction of the radiation propagation. In this case the radiated electric field has the same orientation as the current and is described by the inhomogeneous wave equation. We do not assume any predefined temporal dependence of the field but assume the dominant propagation along the x axis, $E = F(t,x,y,z)e^{-ikx}$, that is typical for solving problems in the paraxial approximation for a slow varying envelope, $\left|F''_{xx}\right| << \left|2ik_x F'_x\right|$.

Writing the current in the spectral representation over plane waves running in the lateral (y,z) plane

$$j = j_0\delta(x - x_0)\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi(\omega)e^{i\omega(t - q_1 y - q_2 z)}d\omega \qquad (1)$$

and using the Green's function method we obtain the spectral representation of the radiated field

$$E = j_0\frac{\mu_0\mu}{2ik}\theta(x - x_0)e^{i(x-x_0)\frac{q^2-k^2}{2k}}\times$$
$$\frac{\partial}{\partial t}\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\Phi(\omega)e^{-i\omega^2\frac{m(x-x_0)}{2kv^2}+i\omega(t+i\frac{\mu_0\mu\sigma}{2k}(x-x_0)-q_1 y-q_2 z)}, \qquad (2)$$

where $v = c/\sqrt{\varepsilon\mu\varepsilon_0\mu_0}$ is the velocity of light in the medium and the coefficient $m = 1 - q^2v^2$, $q^2 = q_1^2 + q_2^2$ shows the degree of paraxial approximation.

II. TIME-SPATIAL DRIFT OF PULSES

A. *Decelerating Airy pulse*

The odd spectrum function $\Phi(\omega) = \exp[i(\omega T + i\alpha)^3/3]$ provides the radiated wave with the envelope described by the Airy function:

$$E(t,x) = \frac{i\mu_0\mu}{2kT}j_0 e^{-ik(x-x_0)/2}\theta(x - x_0)\varphi(x)$$
$$\frac{\partial}{\partial t}\left\{\exp\left[i\left((\tau + i\frac{\mu_0\mu\sigma}{2k}(x - x_0))\right)\frac{1}{T}(m\frac{x - x_0}{2kv^2T^2} - i\alpha)\right]\right.$$
$$\left.\mathrm{Ai}\left[\left(\tau + i\frac{\mu_0\mu\sigma}{2k}(x - x_0)\right)\frac{1}{T} - \left(m\frac{x - x_0}{2kv^2T^2}\right)^2 + 2i\alpha m\frac{x - x_0}{2kv^2T^2}\right]\right\} \qquad (3)$$

Here, $\tau = t - q_1 y - q_2 z$, T is a normalized parameter, the parameter α ensures energy finiteness of the source [2] and the function φ is equal to

$$\varphi(x) = i\frac{2}{3}\left(m\frac{x - x_0}{2kv^2T^2}\right)^2 - 2\alpha\left(m\frac{x - x_0}{2kv^2T^2}\right)^2 + i\alpha^2 m\frac{x - x_0}{2kv^2T^2}$$

The trajectory of the Airy pulse envelope in time-spatial coordinates (t, x, y, z) lies on the surface of a parabolic cylinder $(t - q_1 y - q_2 z)\frac{1}{T} - \left(m\frac{x - x_0}{2kv^2T^2}\right)^2 = const$, Fig. 1. The pulse envelope moves on this surface along the longitudinal axis x decelerating and drifts uniformly with time in the lateral direction. The velocity of the envelope movement in the longitudinal direction is given as a function $\dot{x} = 2k^2v^4T^3m^{-2}/(x - x_0)$ of the distance from the source or as a function of time $\dot{x} = kv^2T^{3/2}m^{-1}/\sqrt{t - const\,T}$. This velocity tends to zero with time as well as with the distance from the source. The acceleration of this movement $\ddot{x} = -\dot{x}^2/(x-x_0)$ is negative everywhere in the region of the pulse existence, $x - x_0 > 0$, and it also tends to zero with the distance from the source confirming the decelerating character of the movement. The trajectories of the envelope in normalized time-spatial coordinates $\tau = t/T$, $\xi = x/vT$ is shown in Fig. 2.

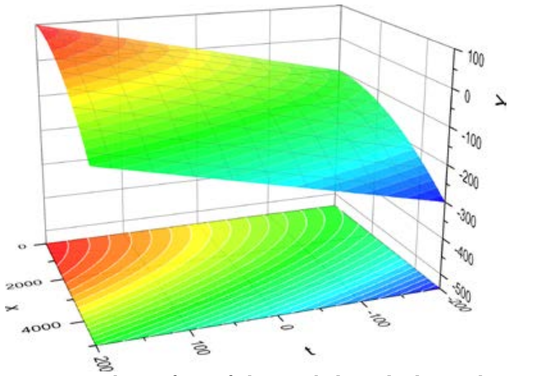

Fig. 1. The surface of the parabolic cylinder in the time-spatial space along which the Airy pulse propagates.

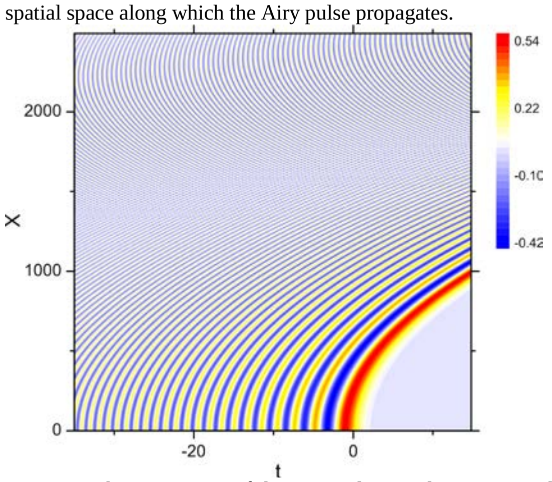

Fig. 2. The trajectories of the Airy pulse envelope generated by the source.

Note that the vertex of the parabolic trajectory is situated at the source point therefore the problem with the backward flow of time does not appear.

B. Decelerating Gaussian

The even spectrum function $\Phi(\omega) = \exp[-(\omega T)^2/4w^2]$ corresponding to the running in the transverse plane Gaussian source current gives the field of the running Gaussian pulse. If the medium is non-dissipative, $\sigma = 0$, then the expression for the field is significantly simplified:

$$E = \frac{j_0\mu_0\mu}{2ik}\frac{w\theta(x-x_0)}{2T\sqrt{\pi}\sqrt[4]{1+m^2u^2(x-x_0)^2}}\times \frac{\partial}{\partial t} e^{-\frac{w^2}{T^2}\frac{\tau^2}{1+m^2u^2(x-x_0)^2} - ik(x-x_0)/2 + i\left(\frac{w^2}{T^2}\frac{\tau^2 mu(x-x_0)}{1+m^2u^2(x-x_0)^2} - \frac{1}{2}\arctan(mu(x-x_0))\right)} \quad (4)$$

where $u = \frac{2w^2}{kv^2T^2}$. Considering the index of the first exponent in (4) we obtain the equation for the surface of the hyperbolic cylinder type $(t - q_1 y - q_2 z)^2 const^{-1} - m^2u^2(x-x_0)^2 = 1$, Fig. 3, on which the pulse envelope moves. The envelope movement is decelerating along the longitudinal axis x with the velocity $\dot{x} = t\left[const\, m^2u^2(x-x_0)\right]^{-1}$. In contrast to the Airy pulse this velocity asymptotically tends to the nonzero value $\dot{x}_\infty = 1/\sqrt{m^2u^2 const}$ with time as well as with the distance from the source.

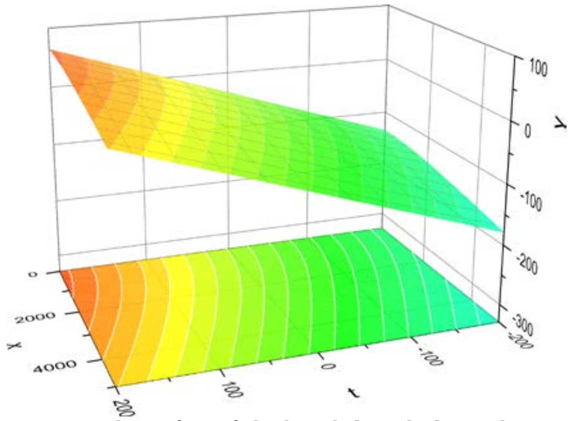

Fig. 3. The surface of the hyperbolic cylinder in the time-spatial space along which the Gaussian propagates.

III. CONCLUSION

A time dependent electromagnetic field in paraxial approximation is generated in the form of Airy or Gaussian pulses. It is shown that these pulses propagate in time with deceleration along the dominant propagation direction and drift uniformly in time and in the lateral direction. The velocity of the envelope tends to zero with time and distance from the source for the Airy pulse and to an asymptotic nonzero value for the Gaussian.

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