QUASI-INTERMITTENCY OF WAVES AND THEIR COMPLEXITY IN MODULATED DIELECTRIC MEDIUM

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***Abstract* - A correlation between the Hurst's index of an electromagnetic signal and its complexity in time-varying medium is shown.**

I. INTRODUCTION

Parametric phenomena in active media have been attracting much attention for a long time in connection with the generation and amplification of electromagnetic waves or by the time variation of the medium parameters. In systems with distributed parameters the main features of the wave transformation by the medium nonstationarity can be revealed when a simpler law changes the medium parameters and an exact solution of the problem can be constructed. In this paper the electromagnetic wave transformation in a medium with parameters that undergo change in a form of a finite packet of periodic rectangular pulses is considered. Regularities in the transformation are quantified by two characteristics: the Hurst's index [1] and the complexity [2].

II. WAVE TRANSFORMATION UNDER MEDIUM MODULATION

We consider an unbounded dielectric dissipative medium, the permittivity and conductivity of which are modulated according to the law of a finite packet of N rectangular periodic pulses

$$\varepsilon(t) = \varepsilon + (\varepsilon_1 - \varepsilon)\sum_{k=1}^{N}\left\{\theta(t-(k-1)T) - \theta(t - T_1 - (k-1)T)\right\}$$
$$\sigma(t) = \sigma_1\sum_{k=1}^{N}\left\{\theta(t-(k-1)T) - \theta(t - T_1 - (k-1)T)\right\} \tag{1}$$

Here, $\theta(t)$ is the Heaviside unit function, T is the duration of the period of the parameters change, T_1 is the duration of the disturbance interval, and all time variables are normalized to some wave frequency ω, $t \to \omega t$. The modulation commences at zero moment of time, before which there exists a plane harmonic wave $E_0(t,x) = \exp[i(t-kx)]$ as an initial field in the undisturbed medium. Each time the jump of the medium properties changes the electromagnetic field. We designate the field on the disturbance interval as E_n, the field on the inactivity interval as F_n.

After the beginning of the modulation by the disturbance interval the initial wave splits into two, forward and backward, waves $E_1 = \exp(-st - ikx)\left[C_1 \exp(iq\,t) + D_1 \exp(-iq\,t)\right]$ with new amplitudes and new frequency $q = (a^2 - s^2)^{\frac{1}{2}}$ where $a^2 = \varepsilon / \varepsilon_1$, $s = \sigma_1 / \omega\varepsilon_0\varepsilon$, and ε_0 is the vacuum permittivity. On the remaining undisturbed interval of the first period of modulation the field splitting remains, $F_1 = \exp(-ik)\left[A_1 \exp(i\,t) + B_1 \exp(-i\,t)\right]$, but the frequency returns to the primary one.

The field on the other disturbance intervals also consists of two, direct and inverse, waves $E_n = \exp(-st - ikx)\left[C_n \exp(iq\,t) + D_n \exp(-iq\,t)\right]$ of changed frequency while the field on the inactivity intervals consists of two waves $F_n = \exp(-ikx)\left[A_n \exp(i\,t) + B_n \exp(-i\,t)\right]$ but of the unchanged frequency.

The relations between the direct and the inverse secondary wave amplitudes $w_N = D_N e^{-i2(N-1)qT} / C_N$ on the disturbance intervals and $p_N = B_N e^{-i2NT} / A_N$ on the inactivity intervals are governed [3] by the sequence

$$r_{N+1} = 4u^2 / (4u^2 - r_N) \tag{2}$$

and the generalized parameter

$$u = \cos(qT_1)\cos(T - T_1) - \frac{a^2 + 1}{2q}\sin(qT_1)\sin(T - T_1)\,. \qquad (3)$$

III. Hurst's index

The ratio between the amplitudes of the forward and backward waves, governed by the sequence r_n, can have regular or irregular behaviour depending on the generalized parameter *u*.

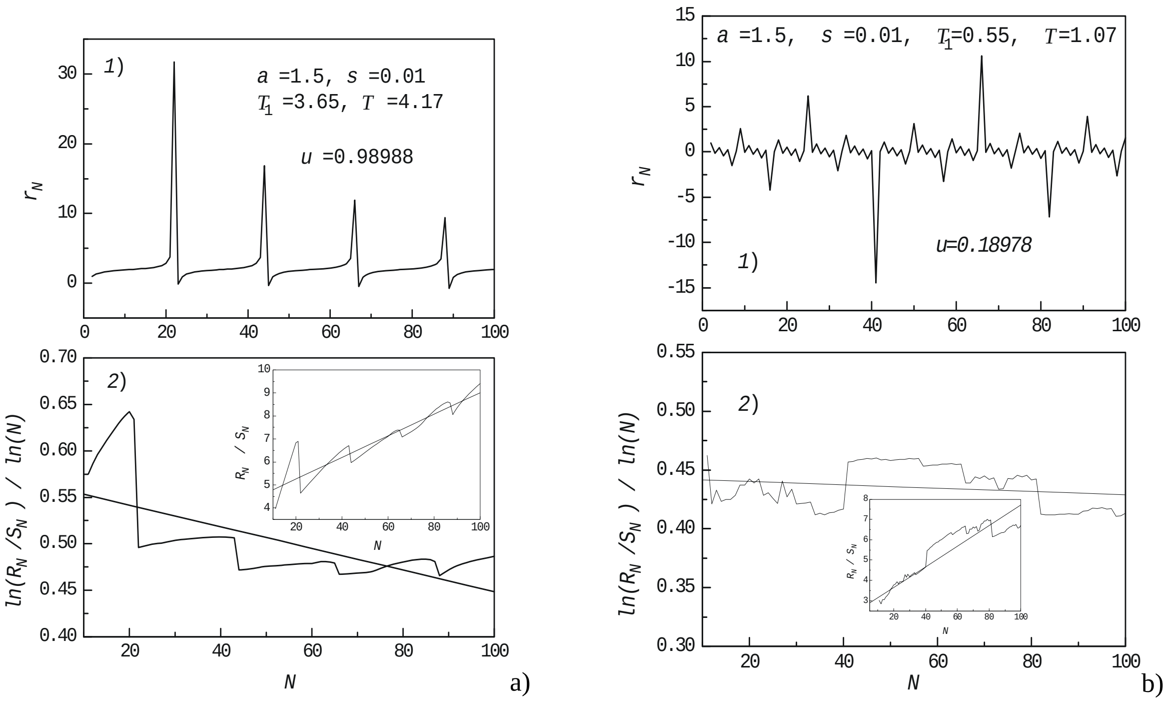

Fig. 1. Near regular, a), and irregular, b), behaviour of the sequence r_N and corresponding Hurst's index.

If $u^2 < 1$, the sequence r_N has irregular character that is there are long intervals in the sequence of the modulation periods where r_N changes almost regularly. After which the relatively short intervals of strong irregular behaviour of r_N take place. This phenomenon can be termed "quasi-intermittency". The presence of the quasi-intermittency can be confirmed by the Hurst's method [3], according to which the time series of r_n is characterised by the Hurst's index H in the asymptotic behaviour of the function

$$H \sim \ln(R_n / S_n)/\ln n \qquad (4)$$

where $R_n = \max_{1\le k\le n} X(k,n) - \min_{1\le k\le n} X(k,n)$,

$$X(k,n) = \sum_{i=1}^{k}\left(r_i - \langle r\rangle_n\right),\ \langle r\rangle_n = \frac{1}{n}\sum_{i=1}^{n} r_i,\quad S_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(r_i - \langle r\rangle_n\right)^2}\,.$$

For white noise (a completely uncorrelated signal) this index equals to $H = 0.5$. The value $H > 0.5$ $(H < 0.5)$ is associated with the long-range correlation when the time series exhibits persistence (antipersistence).

The analysis of the sequence r_N in our case shows that the change of the amplitude ratio is associated with the uncorrelated process when $u^2 < 1$. For example, when $u \approx 0.98988$ the index H decreases from ~0.55 to ~0.45, that is illustrated by Fig. 1a. When u^2 decreases the behaviour becomes more irregular, see Fig. 1b for $u = 0.18978$.

IV. Signal complexity

The r_N behaviour can also be characterised by a complexity measure [2]. This measure of complexity shows how much information is stored in the signal and how much information is needed to predict the next value of the signal if we know all the values up to some moment in time. In two limiting cases, when a signal has constant value at all times and when the signal is completely random, a complexity is equal to zero in this framework because of no information about the previous evolution needed to predict the signal in both cases. All intermediate cases have a finite, non-zero value of complexity.

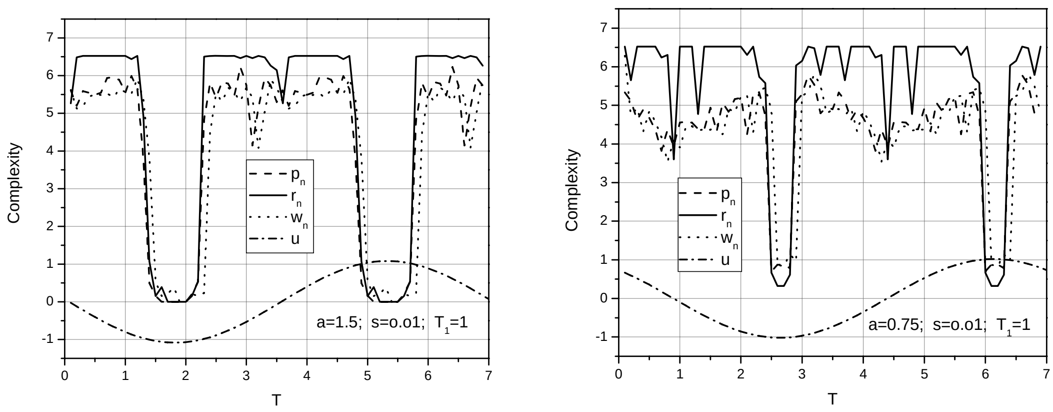

Fig. 2. The behaviour of the complexity vs the modulation periods for various changes of the permittivity.

The algorithm of computing the finite statistical complexity [4] follows the method originated in the works by Crutchfield and others and it consists of considering the symbolic subsequences that form the dynamical 'states' of the system and the time evolution, which is described as transitions between these states with some probabilities P_i. The finite statistical complexity is calculated by the formula:

$$C = -\sum_i P_i \log_2 P_i \qquad (5)$$

The dependence of this measure on the modulation period shows a correlation between the complexity and the Hurst's index *H*, Fig. 2. It is seen that the complexity drops to zero when the parameter *u* (the dash-dot sine-like line) becomes greater than 1. In this case the value of the H index is typical for regular behaviour.

V. Conclusion

Quasi-intermittency that occurs in the wave transformation under time changing of the medium properties can be described by two characteristics, the Hurst's index and the complexity measure. It is shown that these two characteristics correlate with each other and they correlate with the generalized parameter, which control the process of the wave transformation.

References

[1] Hurst H E, Black R P and Simaika Y M, 1965 Long-Term Storage: An Experimental Study (London: Constable)

[2] Crutchfield J. P. and K. Young, Phys. Rev. Lett., **63**, pp. 105-108, 1989.

[3] Nerukh A.G., J. of Physics D: Applied Physics, **32**, pp. 2006-2013, 1999.

[4] D. Nerukh, G. Karvounis, and R. C. Glen, J. Chem. Phys., **117**(21), pp. 9611-9617, 2002.