# Changing of complexity and spectrum of different signals in time-varying medium

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**Abstract:** -Transformation of a complexity and spectra of electromagnetic signals with different initial complexities in a time-varying medium is investigated for various laws of the medium permittivity change. Three groups of signals with different initial shape and complexity are considered. Formation of the different signal spectrum in dependence on the medium change laws and its connection to the signal complexity calculated using the Volterra integral equation method is shown.

When an electromagnetic signal propagates in a time-varying medium it undergoes a transformation that depends on both the law of the medium change and the initial characteristics of the signal. This transformation leads to a distortion of the signal shape as well as its spectrum. An objective characteristic of such transformation is the 'statistical complexity' of an electromagnetic signal, which is a measure of its informational content [J. P. Crutchfield, K. Young, Phys. Rev. Lett., 63, 105-108, 1989]. The present work is devoted to the investigation of influence of a medium change law on the spectrum and complexity of signals with various initial shape and complexity. The temporal change of the medium permittivity is described by a sequence of step functions that allows obtaining exact solutions for the signal transformation using the Volterra integral equation method. The influence of the subsequent sharp changes of the medium in time in the form of rectangular pulses on the complexity of the electromagnetic signals has been investigated earlier in [Ruzhytska N.N., A.G.Nerukh, D.A.Nerukh, Optics and Quantum Electronics, 35, 347-364, 2003]. The investigation of the signal shape transformation caused by the medium permittivity changes is implemented and calculations are performed exactly for the signals, their spectra and complexities using Crutchfield's 'computational mechanics' approach. The advantage of using the statistical complexity measure is that it quantifies the information contained in the signal. However, contrary to more traditional measures, for example Fourier spectrum, it does not assume the existence of the predefined patterns. Instead, it searches for any possible patterns which, if present, contributed to the informational richness of the signal.

The first group of the signals contains a hump-like and Gaussian-like forms and this group represents signals of a "soft" transformation. The initial complexity of the hump-like signal differs significantly from that of the Gaussian-like one. The Gaussian signals become more complicated with the medium change but this complication has a different character for various laws of the medium change. When the permittivity changes according to the periodic law $\varepsilon = 1.1 + 0.2\sin(\pi n/2)$ the signal shape distortions are not as strong as in the case of the monotonic rise of the permittivity, $\varepsilon = 1 + 0.1n$. The complexity of all signals changes in parallel but in the former case there is a saturation effect while in the latter the complexity increases monotonically. This is also reflected in the behavior of the signal's spectrum, which changes slightly in the case of the periodic medium variations and very significantly when the permittivity grows monotonically. In both cases there is an amplification of the signal but in the latter case a sharpening of the spectrum also occurs. Similar transformations are found for the signals of another kind, which are described by the Laguerre polynomials of the various orders $L_n$. These signals differ slightly from each other so as their complexities. Nevertheless the complexity increases with the order of the Laguerre polynomial.

More distortion of the spectrum is found for the signals of the Lorentz-like form, which can be called as signals of a "hard" transformation. The periodic medium change has very little influence on the spectrum, only uniform amplification of the whole spectrum by the factor of ten occurs during 20 cycles of the medium changes. Contrary to that a significant distortion of the spectrum is present in the case of the monotonic medium change. Spectrum rebuilding begins at about the tenth cycle of the medium change when the permittivity doubles approximately. Then the amplification of some spectrum lines moves to the region of high frequencies and the sharpening of the spectrum becomes apparent and even more pronounced than for the Gaussian-signals. The amplification of some spectrum lines is significantly greater (up to approximately $10^6$ times) after the three-fold increase of the permittivity value.

**Conclusions:** Exact calculations of spectra and complexity for three groups of signals with different initial shapes and complexity are performed. It is shown that the growth of the signal complexity is accompanied by sharpening of the signal spectrum and it reveals more strongly in the case of the permittivity monotonic rise when the spectrum becomes distinctly striped than in the case of the periodic medium changes.