Reflection of a spatial-temporal Airy pulse from a layer

A. Nerukh[1], *Senior Member, IEEE*, D. Zolotariov[1], D. Nerukh[2], T. Benson[3], *Senior Member, IEEE*
[1]*Kharkov National University of Radio Electronics, 14 Lenin Ave., Kharkov, 61166 Ukraine*
[2]*Non-linearity and Complexity Research Group, Aston University, Birmingham, B4 7ET, UK*
3George Green Institute for Electromagnetics Research, University of Nottingham, University Park, Nottingham, NG7 2RD, UK

ABSTRACT
The interaction of an Airy pulse with a dielectric layer is investigated theoretically. Approximate analytical expressions for reflected and transmitted waves are derived in the form of Taylor series. These series consist of shifted Airy pulses which are decelerated in time and space and deceleration becomes stronger with a number of a term of series.
Keywords: decelerating Airy pulse; Airy pulse reflection and transmission.

1. INTRODUCTION

Very intriguing properties of optical Airy beams serve as motivation for recent active development in the theory and experimental applications of these beams [1-7]. The detailed analysis of the mathematical aspects as well as physical interpretation of electromagnetic Airy beams was done by considering mostly the wave as a function of spatial coordinates only and assuming that their time dependence is harmonic. Only a few papers consider a more general temporal dependence [8-12] where a parabolic relation exists between temporal and spatial variables. Appearance of soliton shedding from Airy pulse in Kerr medium is shown in [11] where the nonlinear paraxial equation in time domain is investigated numerically. An important physical difference between spatial and temporal acceleration is analyzed in [12]. However, for our knowledge, propagation of Airy pulses is considered in homogeneous medium only whereas any optical system suggests presence various obstacles, such as lenses, reflectors, mirrors and so on. Therefore it is important to investigate a physical phenomenon of pulse interaction with such an object. We consider here reflection and transmission of the Airy pulse by the simplest object, a dielectric layer.

2. GENERATION OF AN AIRY PULSE

We consider an electromagnetic pulse generated by a current oriented perpendicularly to a direction of a pulse propagation, $\mathbf{j} = (0, j(t,x), 0)$. The electric field of the pulse has the same orientation $\mathbf{E} = (0, E(t,x), 0)$ and satisfies the 1D equation

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} - \mu_0 \sigma \frac{\partial E}{\partial t} = \mu_0 \frac{\partial j}{\partial t}, \tag{1}$$

where $v = 1/\sqrt{\varepsilon\varepsilon_0\mu_0}$, ε is the relative medium permittivity, ε_0, μ_0 are the permittivity and the permeability of vacuum respectively and σ is the medium conductivity. Considering the wave $E = F(t,x)e^{-ikx}$ in paraxial approximation $\left|F''_{xx}\right| << \left|2ik_x F'_x\right|$ we find the equation for the envelope

$$-2ik\frac{\partial F}{\partial x} - k^2 F - \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} - \mu_0 \sigma \frac{\partial F}{\partial t} = \mu_0 \frac{\partial j}{\partial t} e^{ikx}, \tag{2}$$

which can be classified as a parabolic equation known in the theory of thermal conductivity and diffusion. The fundamental solution to (2) (the Green's function) can be constructed by virtue of the Fourier transform:

$$G_p = \frac{(1+i)v}{4\sqrt{\pi k x}}\theta(x) e^{i\frac{k}{2}x + i\frac{kv^2}{2x}\left(t + i\frac{\mu_0 \mu \sigma}{2k}x\right)^2} \tag{3}$$

where $\theta(x)$ is the Heaviside unit function.

Let the source $j(t,x)$ of an electromagnetic wave be located at a point x_0 and represented by temporal dependence in the form of the Airy function, $j = \delta(x - x_0) v \mathrm{Ai}\left(vt/L\right) e^{\alpha vt/L}/L$. Here, L is a width of a dielectric layer and α provides finite energy of the source (as it was proposed in [1] for a stationary problem). Using the Green's function (3) we obtain the electromagnetic wave generated by this source

$$E_0(t,x)=\frac{-iv\mu_0}{2kL}e^{-\frac{ik(x-x_0)}{2}}\frac{\partial}{\partial t}\mathrm{Ai}\left(2i\alpha X-X^2+\left(\frac{vt}{L}+i\mu_0\sigma vLX\right)\right)\times$$
$$\exp\left[i\left(\alpha^2X-i\alpha\left(\frac{vt}{L}+i\mu_0\sigma vLX\right)+2i\alpha X^2-\frac{2}{3}X^3+\frac{x-x_0}{2kL^2}\left(\frac{vt}{L}+i\mu_0\sigma vLX\right)\right)\right] \quad (4)$$

where $X=\dfrac{x-x_0}{2kL^2}$. The movement of this pulse envelope given by the Airy function is decelerated that follows from the expression for its velocity

$$\frac{dx}{dt}=\frac{2k^2vL^3}{x-x_0-i\alpha 2kL^2}=\frac{kvL}{\sqrt{vt/L-const-\alpha^2}}\,. \quad (5)$$

The velocity decreases with distant and time and the pulse stops at infinity. The effect of the deceleration is the result of the paraxial approximation. It is absent in the exact solution of the wave equation (1), which has the form of the Airy pulse $E=v\mu_0\mathrm{Ai}(t-|x-x_0|/v)/2$ also but moves uniformly with constant velocity.

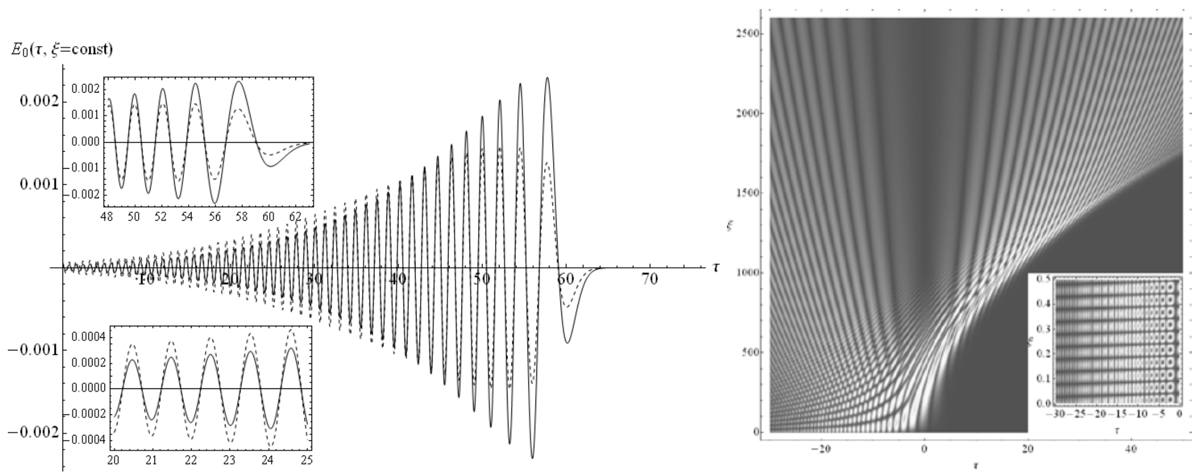

Figure 1. (left) The Airy pulse $E_0(\tau,\xi)$ at the points: $\xi=0$ (solid), $\xi=-4$ (dashed); (right)the trajectories of the envelope constant values.

The temporal behaviour of the Airy pulse and its trajectory are illustrated by Fig. 1. Here and further the dimensional variables are used: $\tau=vt/L$, $\xi=x/L$.

3. INTERACTION OF THE AIRY PULSE WITH A DIELECTRIC LAYER

We investigate the interaction of the Airy pulse with a plane dielectric layer with the permittivity ε_1 located in the interval $[0,L]$ when the pulse falls normally onto the layer. We proceed from the formula for the wave (4) written in the form of double Fourier expansion with respect to time and spatial variables:

$$E_0(t,x)=-\frac{\mu_0}{8k\pi^2}\cdot\frac{\partial}{\partial t}\int_{-\infty}^{\infty}e^{\frac{i(\omega T+i\alpha)^3}{3}}\int_{-\infty}^{\infty}\frac{e^{i(\omega t+\eta(x-x_0))}}{\frac{k}{2}+\frac{\omega^2}{2kv^2}+\eta-i\omega\frac{\mu_0\mu\sigma}{2k}}d\eta d\omega\,. \quad (6)$$

3.1 The reflection of the Airy pulse from a layer

The wave reflected from the layer is given by the integral

$$E_R(t,x)=-\frac{\mu_0}{8k\pi^2}\cdot\frac{\partial}{\partial t}\int_{-\infty}^{\infty}e^{\frac{i(\omega L/v+i\alpha)^3}{3}}\int_{-\infty}^{\infty}\frac{r\left(1-e^{-i2\sqrt{\varepsilon_1}L\eta}\right)}{1-r^2e^{-i2\sqrt{\varepsilon_1}\eta}}\frac{e^{i(\omega t-\eta(x-x_0))}}{\frac{k}{2}+\frac{\omega^2}{2kv^2}-i\omega\frac{\mu_0\mu\sigma}{2k}+\eta}d\eta d\omega\,. \quad (7)$$

where the reflection coefficient for a plane wave is used and $r=(\sqrt{\varepsilon}-\sqrt{\varepsilon_1})/(\sqrt{\varepsilon_1}+\sqrt{\varepsilon})$. Expansion of the reflection coefficient in (7) into a Taylor series with respect to $r<1$ gives the reflected field

$$E_R\left(t,x\right)=-\frac{\mu_0 v^2 i}{2\kappa L}\cdot\sum_{N=0}^{\infty} r^{2N+1}\left[E_N\left(t,x\right)-E_{N+1}\left(t,x\right)\right], \tag{8}$$

which consists of the shifted Airy functions

$$E_N\left(t,x\right)=e^{i\left(\left(I_N-i\alpha\right)\frac{vt}{L}-\frac{2}{3}I_N{}^3+2i\alpha I_N{}^2+(\alpha^2-k^2L^2)I_N\right)}\left[i\left(I_N-i\alpha\right)\mathrm{Ai}\left(\frac{vt}{L}+2i\alpha I_N-I_N{}^2\right)+\frac{\partial}{\partial\tau}\mathrm{Ai}\left(\frac{vt}{L}+2i\alpha I_N-I_N{}^2\right)\right] \tag{9}$$

where $I_N=\left[\left(x-x_0\right)+2\sqrt{\varepsilon_1}LN\right]/2kL^2$ and the medium dissipation is neglected, $\sigma=0$.

The reflected pulse is shown in Fig. 2. It is seen that the reflected pulse decelerates with distant from the layer.

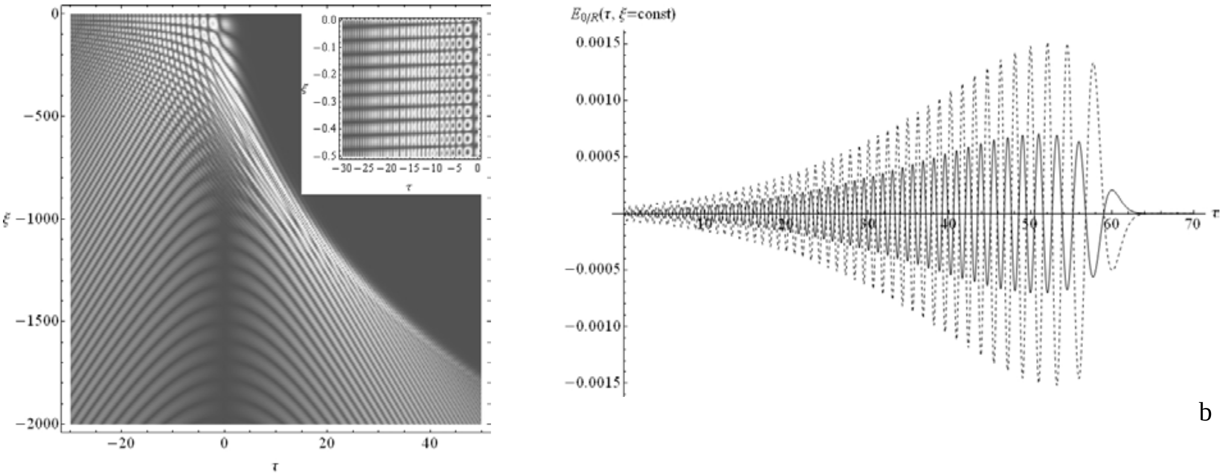

Figure 2. (left) The level lines for the reflected pulse $\left|E_{refl}\left(\tau,\xi\right)\right|$, details near $\xi=0$ in the inset; (right) comparison of the reflected wave (solid) and the initial one (dotted).

3.2 The transmittance of the Airy pulse through a layer

In the same way as for the reflected pulse but usage of the transmittance coefficient we obtain the transmitted wave

$$E_{trans}\left(t,x\right)=-\frac{\mu_0 i}{2kT^2}\left(1-r^2\right)e^{ik\left(\sqrt{\varepsilon}-\sqrt{\varepsilon_1}\right)L}\sum_{N=0}^{\infty} r^{2N}E_N\left(t,x\right), \tag{10}$$

Fig. 3 illustrates the transmitted pulse (left) and comparison it with the initial wave (right).

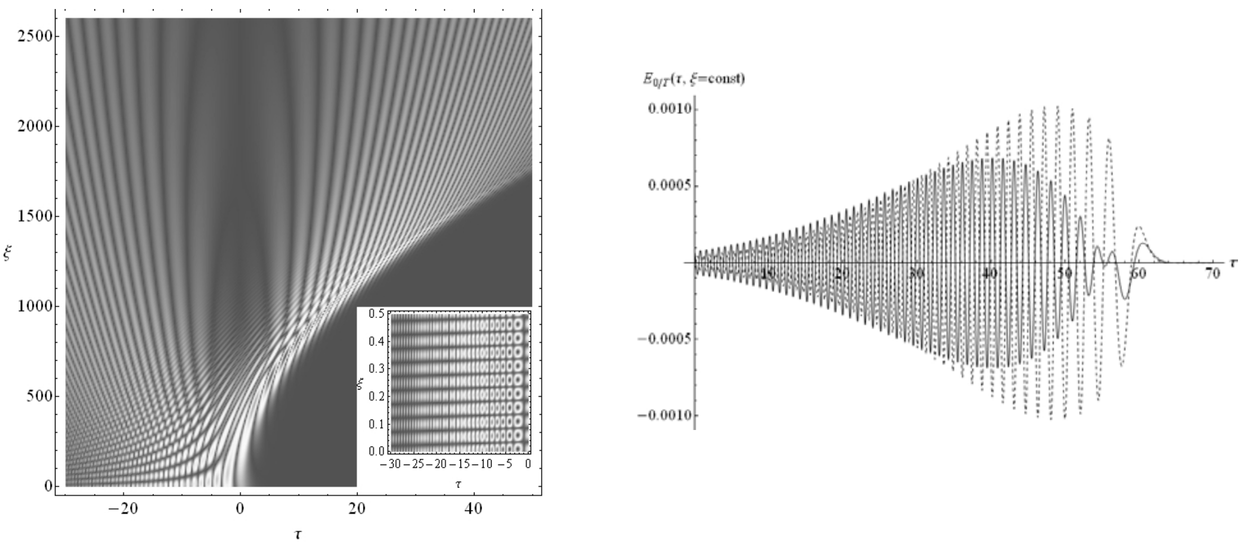

Figure 3. (left) The trajectories of the level lines for the wave $E_{trans}\left(\tau,\xi\right)$ with details near $\xi=0$ in the inset; (right) Comparison of the transmitted pulse with the initial wave at the point $\xi=2$: a solid line is for the initial wave, the dashed one is for the transmitted wave.

Growth of the transmitted wave with respect to the initial one is explained by its shift. If one adds the shift $\Delta\xi = 0.0565$ to the initial wave then the both waves acquire the same phase, Fig. 4 (left), and the amplitude of the transmitted wave becomes less than the initial one, Fig. 4 (right).

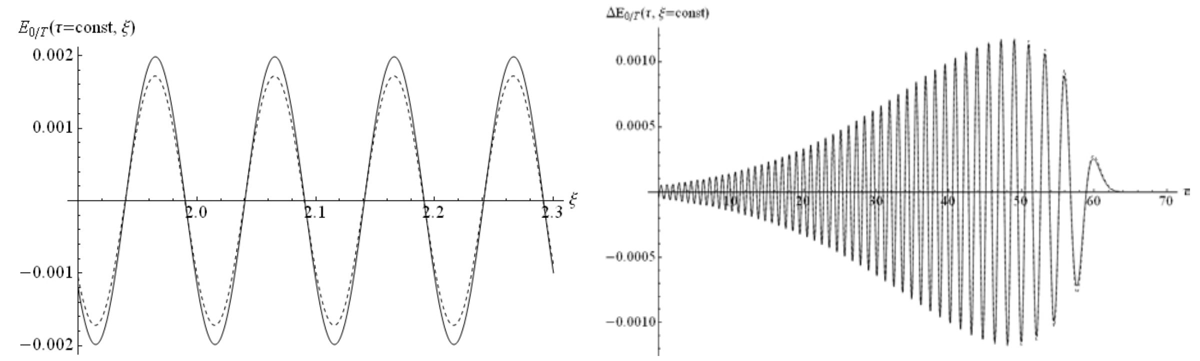

Figure 4. (left) Reduction of the initial wave (solid) and the transmitted one (dashed) to the same phase at the moment $\tau = 5$ by shift of the initial wave in space on $\Delta\xi = 0.0565$; (right) Matching of amplitudes of the initial and transmitted waves.

It is worth to note that the shifted Airy pulses in the series (8) and (10) are decelerated in time and space and deceleration becomes stronger with a number N of a term of series:

$$\frac{dx}{dt} = \frac{2k^2 v L^3}{x - x_0 - i\alpha 2kL^2 + 2\sqrt{\varepsilon_1} LN} \tag{11}$$

4. CONCLUSIONS

The time dependent electromagnetic Airy pulses are obtained as a solution to a paraxial equation with a source using Green's functions in time domain. The Airy pulses propagate with deceleration and stops at the infinite distance from the source. The interaction of the Airy pulse with a dielectric layer is considered and expressions for reflected and transmitted waves are obtained in the form of rapidly convergent series. Each term of the series represents a decelerating Airy pulse which deceleration becomes stronger with a number of a series term.

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