Complexity of Electromagnetic Pulse Passing a Layer of Nonlinear Medium

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ABSTRACT

Application of the Volterra integral equation method to the electromagnetic field interaction with a non-linear medium in a dielectric layer is considered. Transformation of the initial Gaussian pulse and the modulated Gaussian pulse is computed. Complexity of the pulses is calculated using 'statistical complexity' measure and its change in time is analysed.

Keywords: electromagnetic transients, integral equations in time domain, non-linear medium, signal complexity.

1. INTRODUCTION

It is very well known that an interaction of optical pulses with a non-linear medium is of significant importance in quantum electronics and optical communications technology. All these studies concern ultrafast electromagnetic transients in such a medium and their solution in time domain is a subject of extensive research. It is known also that the form of the electromagnetic pulse as well as its complexity changes sufficiently during the interaction with the nonlinear medium. To investigate such non-stationary phenomena one must consider initial value problems. It is convenient to do this by Volterra integral equation method that allows to take into account initial conditions in a natural way and to use numerical the methods for the investigation of he initial problems [1]. In this paper we consider the behaviour of the electromagnetic pulses passing through the layer of a non-linear dielectric medium. In order to estimate how complex the pulses are the 'finite statistical complexity' measure of the pulses is calculated. It is based on the formalism called 'computational mechanics' that was originated in the works by Crutchfield and others [2]. Analysis of the signal complexity in the time modulated linear medium has been investigated earlier in [3].

2. TRANSFORMATION OF THE FIELD IN THE NONLINEAR LAYER

Consider an initial electromagnetic pulse $\bar{E}_0(t,x)$ in a background medium having a relative permittivity e. At some moment of time the pulse enters to a layer that exists in this background medium. The medium inside the layer is described by the polarisation $\bar{P}(t,x)$. In the case of the initial pulse propagating along the normal to the layer, all electromagnetic values depend only on the spatial coordinate x and the time t and the problem becomes a one-dimensional problem. The electromagnetic field inside and outside of the layer is described by the integral equation [4]

$$E(t,x) = E_0(t,x) - \frac{1}{2}\frac{\partial}{\partial t}\int_0^{\infty} dt'\int_0^1 dx'd\left(t - t' - |x - x'|\right)\left[\frac{1}{ee_0}P(t,x) - \frac{e-1}{e}E(t',x')\right]. \qquad (1)$$

This equation is given in dimensionless variables $t = \frac{v}{L}t$ and $x = \frac{x}{L}$ that is convenient for the numerical calculations. Here $v = c/\sqrt{e}$, c is the velocity of light in vacuum, $d(t)$ is the Dirac delta-function, L is the width of the layer, and e_0 is the permittivity of the free space. The electromagnetic field quantities are also normalized: $E(t,x) = \bar{E}(Lt/v, Lx)/\max\left|\bar{E}_0\right|$, $P(t,x) = \bar{P}(Lt/v, Lx)/\max\left|\bar{E}_0\right|$. The medium polarisation consists of two terms, $P(t,x) = P_L(t,x) + P_{NL}(t,x)$, where the linear part describes a medium with the constant permittivity e_1, $P_L(t,x) = e_0[e_1 - 1]\bar{E}(Lt/v, Lx)/\max\left|\bar{E}_0\right|$, and the non-linear part describes the cubic nonlinearity

$$P_{NL}(t,x) = c_{NL}^{(3)}\bar{E}^3(Lt/v, Lx)/\max\left|\bar{E}_0\right| = g\,E^3(t,x)\,. \qquad (2)$$

The normalised dimensionless coefficient of the non-linearity g is related to the susceptibility $c_{NL}^{(3)}$ of the third order material non-linearity as $g = c_{NL}^{(3)}\max\left|\bar{E}_0\right|^2$.

3. EVALUATION OF THE SIGNAL COMPLEXITY

The measure of complexity shows how much information is stored in a signal and how much information is needed to predict the next value of the signal if we know all the values up to some moment in time. In two limiting cases, when a signal has constant value at all times and when it is completely random, the complexity is equal to zero in this framework because of no information about the previous evolution needed to predict the signal in both cases. All intermediate cases have a finite, non-zero value of the complexity. To compute the complexity a continuum signal is converted into a sequence of symbols from a predefined alphabet. The 'symbolic dynamics' is analyzed such that the number of dynamical 'patterns' is extracted. The diversity of these patterns and their interrelations define the resulting complexity. It consists of considering the symbolic subsequences that form the dynamical 'states' of the system and the time evolution, which is described as transitions between these states with some probabilities. The finite statistical complexity is calculated by the formula:

$$C = -\sum_i P_i \log_2 P_i \qquad (3)$$

where P_i is the probability of each dynamical state. Calculation of this probability is made by the procedure given in [5].

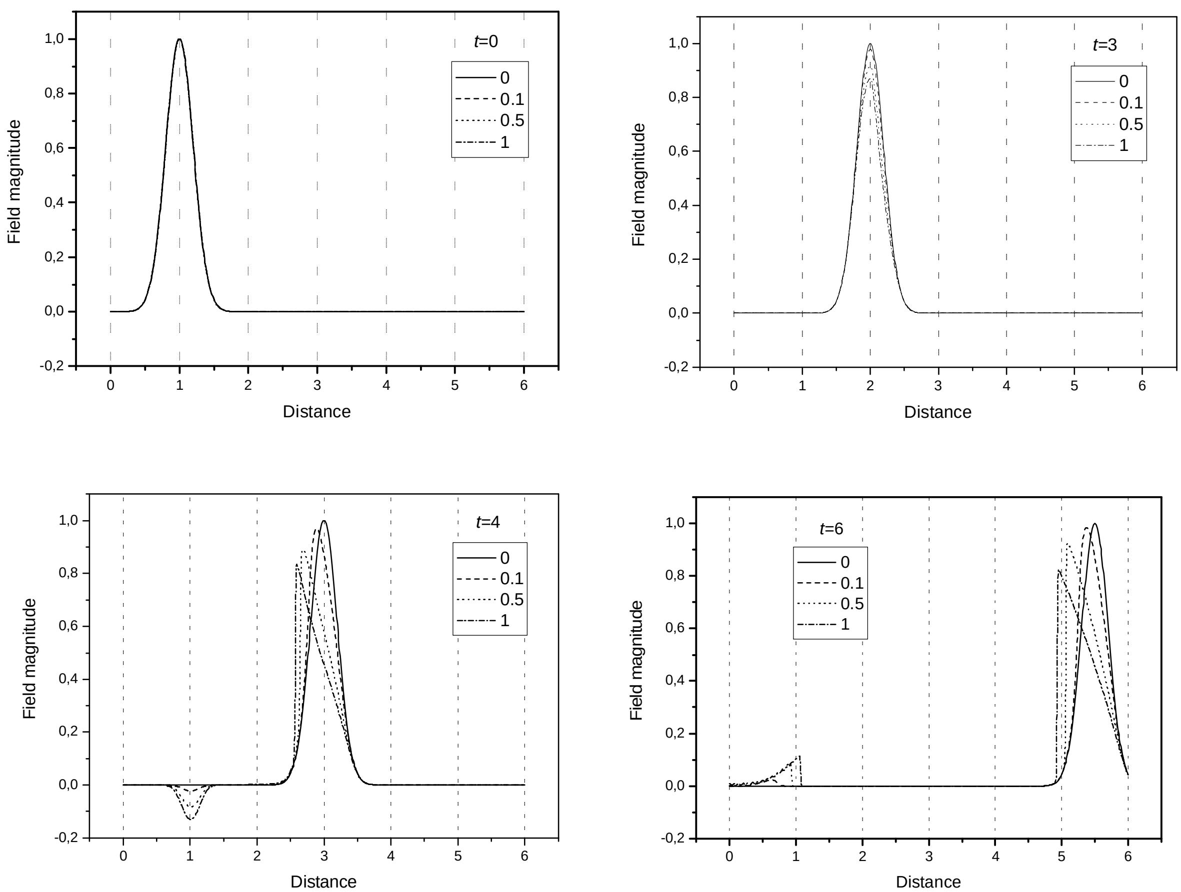


Figure 1. Transformation of the pulse passing through the layer of the nonlinear medium. Snapshots are given for various transformation stages: the initial pulse, t=0; the pulse entering the nonlinear layer, t=3; the pulse leaving the layer, t=4; the pulse far off the layer, t=6.

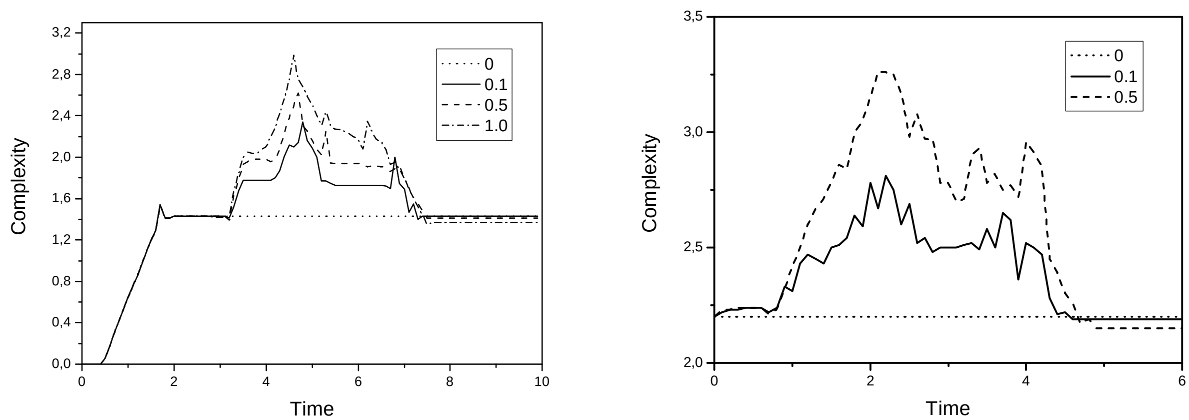
4. RESULTS

When estimating the normalized non-linearity coefficient g it is taken into account that $\max\left|\bar{E}_0\right|^2 \approx I 10^2 \ [V^2/m^2]$ where I is the electromagnetic field intensity. Its value can be equal to $10^{10}[W/m^2]$ the order of magnitudes and can be generated routinely. As the susceptibility $c_{NL}^{(3)}$ can be estimated from $10^{-9} \div 10^{-16}[m^2/V^2]$ for resonance cases to $10^{-19}[m^2/V^2]$ for nonresonance ones then the magnitudes of g can vary in a wide range from 10^3 to 10^{-4}. In this paper the nonlinearity coefficients $g = 0.1$, $g = 0.5$, $g = 1$ are considered. The linear material corresponds to $g = 0$.

The interval [0, 10] in the dimensional measure is taken as the spatial domain, on which the simulations are implemented. The layer of the nonlinear medium occupies the interval [2, 3] and the medium on this interval is characterised by the nonlinearity coefficient g. The linear property of the medium inside the layer as well as outside it is the same and characterised by the permittivity $e = e_1 = 1$.

The pulse of the initial field propagates from the input point at the left boundary of the considered interval and leaves this interval passing through the nonlinear layer on the way. Two pulses for the initial field are considered: the Gaussian pulse $E_0 = \exp\left[-\left(t - t_0 - x\right)^2 / 2s^2\right]$ with the parameters $s = 0.5$ and $t_0 = 2$ and the modulated Gaussian pulse $E_0 = \exp\left[-\left(t - t_0 - x\right)^2 / 2s^2\right]\cos\left[2ph\left(t - x\right)\right]$ with the modulation frequency $h = 2$.

Fig. 1 shows that the pulse starts the transformation at the moment when it enters the nonlinear layer, t=3. At this moment the pulse is reflected from the front surface between linear and nonlinear media. The second reflected pulse appears when the transformed pulse leaves the nonlinear layer, t=4. Further propagation of the pulse does not change its form because the medium is non-dispersive and linear outside the layer.



a) b)

Figure 2. Change of the pulse complexity during passing the nonlinear layer.

During passing through the nonlinear layer the pulse undergoes a strong transformation, which dependence on the nonlinearity coefficient is seen clearly in Fig. 1. The complexity of the pulse changes also but it behaves in more sophisticated manner. A linear rise of the complexity on the initial stage that lasts between zero moment and moment 2 is caused by the pulse entering into the observation domain. Then, on the time interval [2, 3], the pulse is observed entirely but its complexity does not change because it propagates in the linear medium at this period. The main changes in the pulse occur between the moments $t = 3$ and $t = 5$ when it is passing through the nonlinear layer. On this time interval the signal complexity grows and the main contribution to the growth is given by the reflection of the pulse from the layer boundaries, Fig. 2a. The raising of the complexity near the moment 3 is conditioned by appearing of the first reflected pulse when the initial pulse enters the nonlinear medium region. The next raise of the complexity near the moment 5 is conditioned by appearing of the second pulse caused by the reflection from the back front of the nonlinear medium region. When the first reflected pulse goes out of the observation domain the second reflected pulse still remains in the observation domain. So, the complexity falls down and holds a constant value, approximately equal to the complexity value in the period when only the first reflected pulse existed. Further, the second reflected pulse leaves the domain of the observation and the complexity falls down almost to the value of the unchanged pulse. As it is seen from Fig. 2a the complexity of the initial and transformed pulses differs slightly at this final period.

A similar behaviour of the pulse complexity, Fig. 2b, is in the case of the modulated Gaussian pulse $E_0 = \exp\left[-\left(t - t_0 - x\right)^2 / 2s^2\right]\cos\left[2ph\left(t - x\right)\right]$, which shape transformation is shown in Fig. 3. However, in this case the change of the pulse shape is not as significant as in the previous case and the complexity only characterises quantitatively the pulse transformation. One can say that it is impossible to judge by eye how much the shape of the pulse changes. The complexity curve in Fig. 2b is more irregular than that in Fig. 2a that is caused by the fringe-like shape of the modulated pulse,

Fig.3. It provides greater magnitudes of the complexity in the whole domain of the observation and greater difference in the complexity for various values of the nonlinearity coefficient.

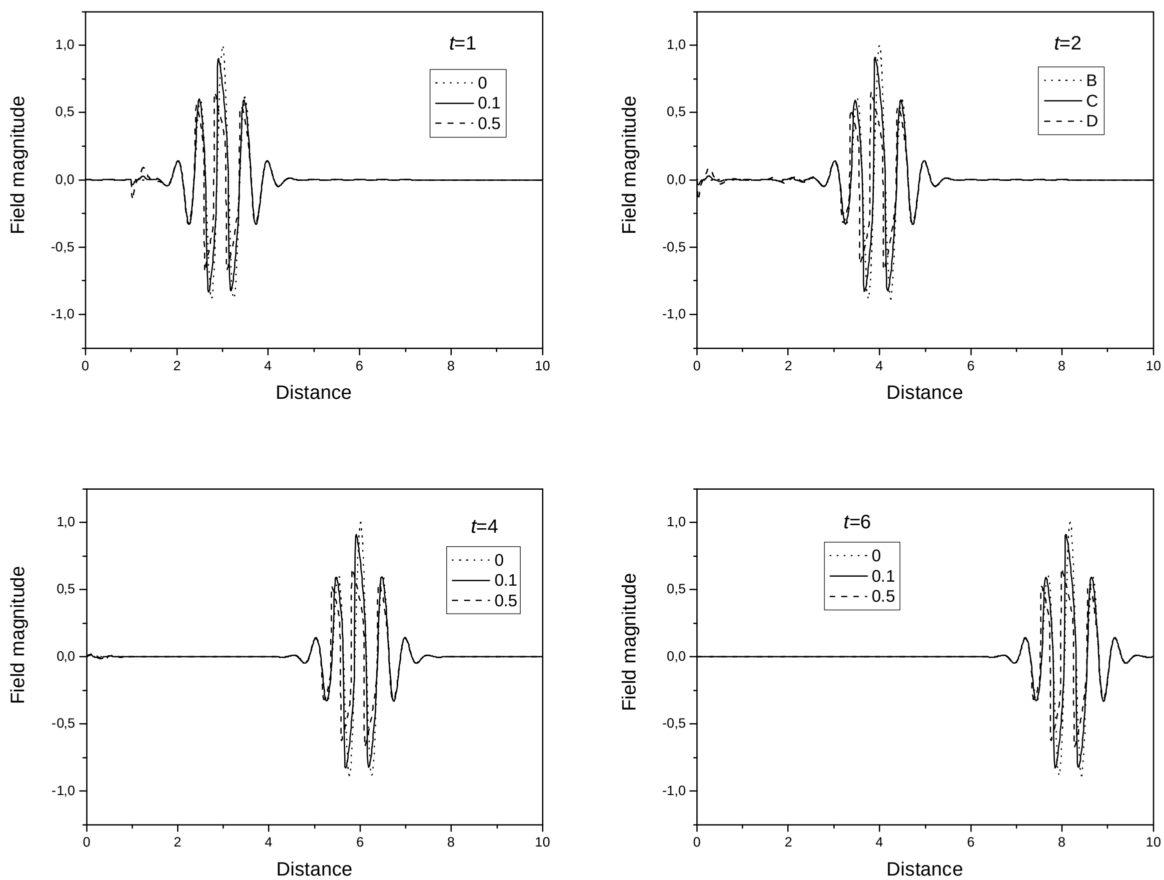


Figure 3. Transformation of the modulated Gaussian pulse passing through the layer of the nonlinear medium.

5. CONCLUSION

Investigations of a transformation of an electromagnetic pulse when it passes through a flat dielectric layer with a nonlinear medium are implemented by the Volterra integral equation time-domain method. This method allows to consider an initial value problem and to compute an electromagnetic field inside as well as outside the layer. The linear part of the permittivity has the same value inside the layer and outside it. Calculations of the pulse complexity show its strong change in the period of passing through the layer. This change is mainly determined by the appearance of the pulses reflected from the layer boundaries. A modulation of the initial pulse does not affect the complexity essentially.

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