Sharpening of the Signal Spectrum with the Change of its Complexity

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ABSTRACT
Transformation of signals with different initial complexities in a time-varying medium is investigated for various laws of the medium permittivity change. Formation of the striped signal spectrum in the monotonically changing medium and its connection to the signal complexity are shown.
Keywords: Electromagnetic transients, signal complexity, time-varying medium.

1. INTRODUCTION

When an electromagnetic signal propagates in a time-varying medium it undergoes a transformation that depends on both the law of the medium change and the initial characteristics of the signal. This transformation leads to a distortion of the signal shape as well as its spectrum. An objective characteristic of such transformation is the 'statistical complexity' of an electromagnetic signal, which is a measure of its informational content [1]. The present work is devoted to the investigation of the influence of various laws of the medium change on the spectrum and complexity of various initial signals propagating in time-varying medium.
The change of the medium permittivity is described by a sequence of step functions that allows to use an exact solutions for the signal transformation caused by a temporal jump change of the medium parameters [2]. The influence of the subsequent sharp changes of the medium in time in the form of rectangular pulses on the complexity of the electromagnetic signals has been investigated earlier in [3].
Three groups of signals with different initial shape and complexity are considered. The investigation of the signal shape transformation caused by the medium permittivity changes is implemented and calculations are performed exactly for the signals, their spectra and complexities using Crutchfield's 'computational mechanics' approach for the latter [4]. The temporal medium variations has the form of a sequence of cycles at each of which the permittivity changes according to various step-wise laws.
The advantage of using the statistical complexity measure is that it quantifies the information contained in the signal. However, contrary to more traditional measures, for example Fourier spectrum, it does not assume the existence of the predefined patterns (*sin* and *cos* in Fourier transformation). Instead, it searches for any possible patterns which, if present, contributed to the informational richness if the signal.

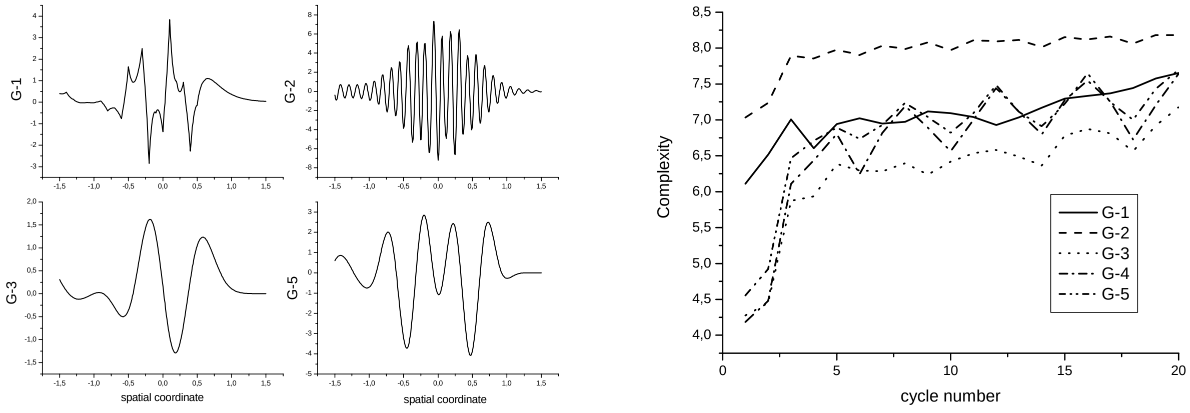
2. SIGNALS OF "SOFT" TRANSFORMATION

The first group of the signals, given in Table 1 together with their initial complexities, contains a hump-like and Gaussian-like forms modulated or not by a harmonic function.

Table 1:

$G_1 = 4\left(e^{-10\|t-x\|} - e^{-5\|t-x\|}\right)$	0.75	$G_2 = 4(e^{-10\|t-x\|} - e^{-5\|t-x\|})\sin 50(t-x)$	0.73
$G_3 = e^{-10(t-x)^2}$	0.42	$G_4 = 4e^{-10(t-x)^2}\left(\sin 11(t-x) - \sin 9(t-x)\right)$	0.45

The complexity of the hump-like signal G_1 differs significantly from that of the Gaussian-like one G_3 and in both cases the modulation by the harmonic function influences slightly the complexity as it is seen for the signals G_2 and G_4.
The Gaussian signals become more complicated with the increase of the cycle number but this complication has a different character for various laws of the medium change. When the permittivity changes according to the periodic law $\varepsilon = 1.1 + 0.2\sin(\pi n/2)$ the signal shape changes are not as strong (Fig. 1a) as in the case of the monotonic rise of the permittivity, $\varepsilon = 1 + 0.1n$, from the value of 1 to 3, Fig. 2a. The complexity of all signals changes in parallel but in the former case there is a saturation effect while in the latter the complexity increases monotonically. This is also reflected in the behavior of the signal's spectrum, which changes slightly in the case of the periodic medium variations, Fig. 3a, and very significantly when the permittivity grows monotonically. In both cases there is an amplification of the signal but in the latter case a sharpening of the spectrum also occurs, Fig. 3b.



a) b)

Figure 1. The transformation of the Gauss-shape signal (a) and its complexity (b) for the periodic medium change.

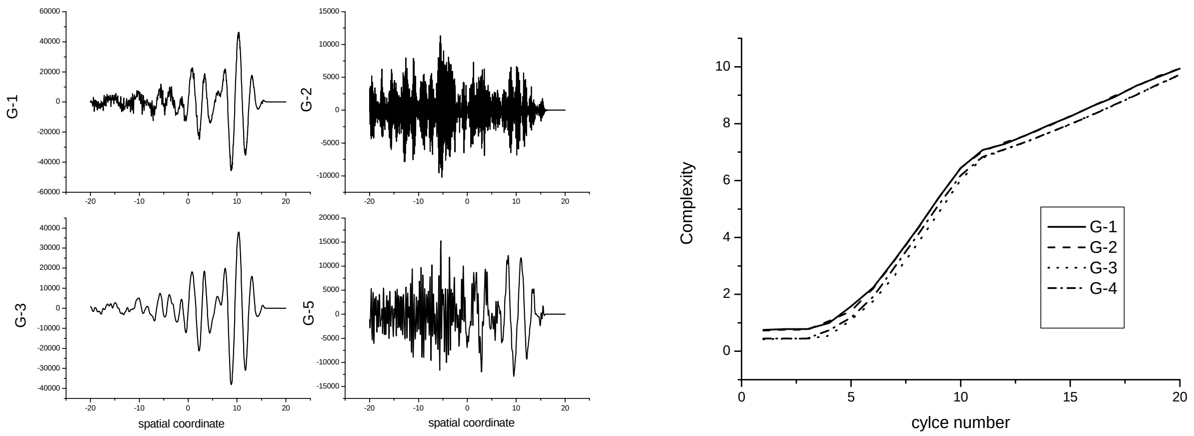


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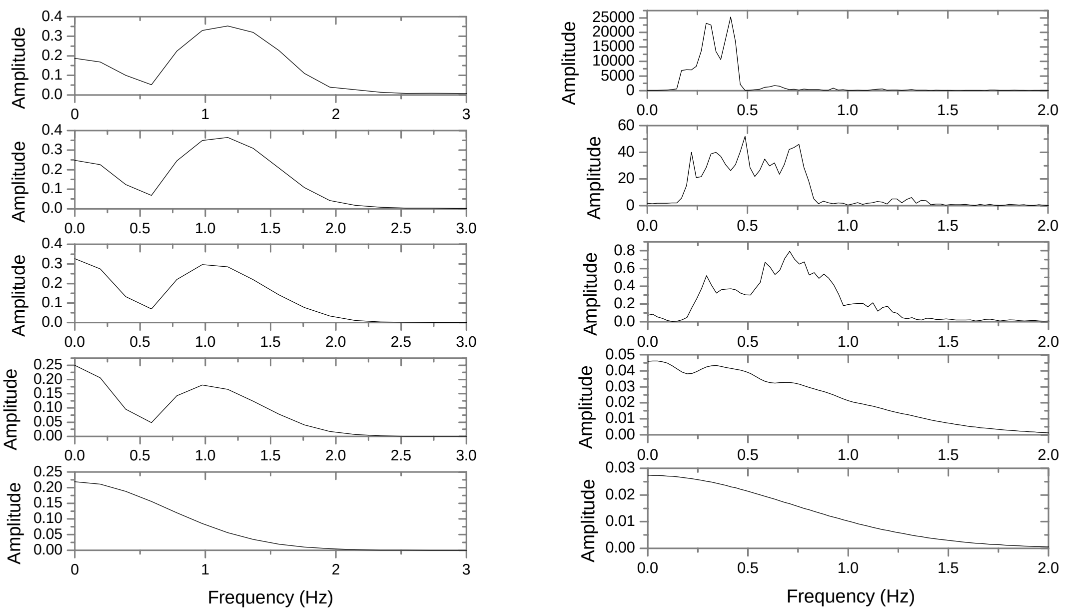
Figure 2. The transformation of the Gauss-shape signal (a) and its complexity (b) for monotonic increase of the medium permittivity.



a) b)

Figure 3. The transformation of the Gauss-shape signal spectrum after 5, 10, 15 and 20 cycles (from bottom to top) of the medium change for the periodic (a) and monotonic rise (b) laws.

Similar transformations are found for the signals of a different kind described by the Laguerre polynomials of the various orders L_n, Table 2.

Table 2:

$Lgr_1 = 3.5e^{-20(t-x)/2} L_1(20(t-x))$	0.45	$Lgr_2 = 1.75e^{-20(t-x)/2} L_2(20(t-x))$	0.55
$Lgr_3 = 0.7e^{-20(t-x)/2} L_3(20(t-x))$	0.60	$Lgr_4 = (1/3.5)e^{-20(t-x)/2} L_4(20(t-x))$	0.64

These signals differ slightly from each other so as their complexities. Nevertheless it is clearly seen that the complexity increase with the order of the Laguerre polynomial.

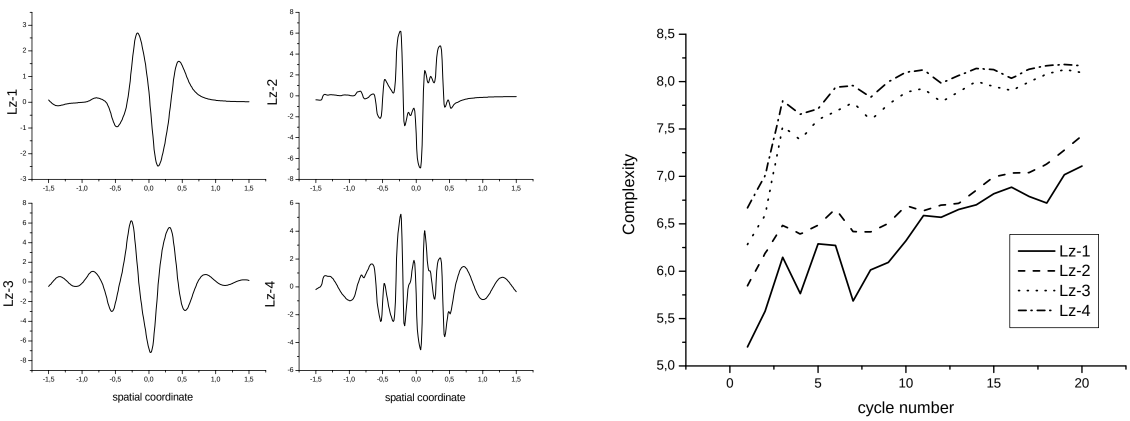
3. SIGNALS OF "HARD" TRANSFORMATION

More distortion of the spectrum is found for the signals of the Lorentz-like form, Table 3. The influence of the harmonic modulation on these signals is more noticeable, (pairs Lz_1 - Lz_3 and Lz_2 – Lz_4, Table 3) and the complexities of the signals Lz_1 and Lz_2 differ significantly.

Table 3:

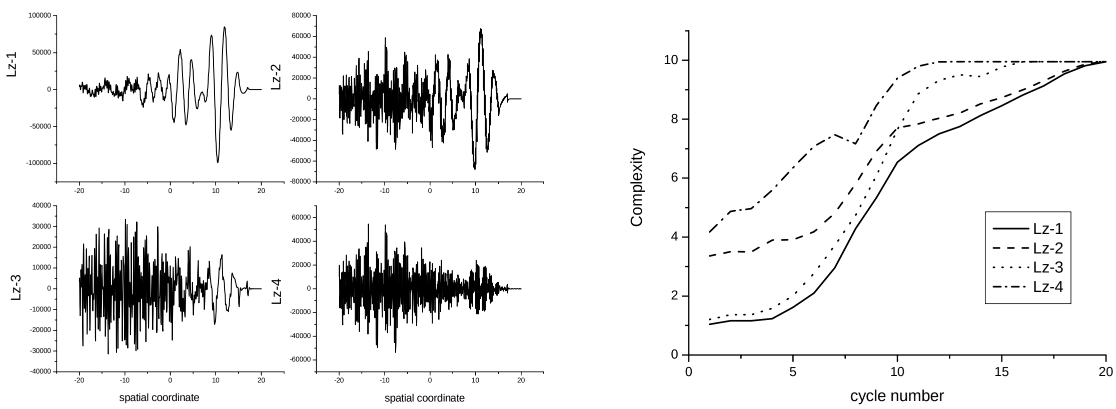
$Lz_1 = 1/(50(t-x)^2+1)$	1.04	$Lz_2 = 75(t-x)/\left(1000(t-x)^2+1\right)$	3.36
$Lz_3 = 1.7\sin 11(t-x)/\left(50(t-x)^2+1\right)$	1.21	$Lz_4 = 75(t-x)\cos 11(t-x)/\left(1000(t-x)^2+1\right)$	4.18

The signal shape changes gradually when the permittivity changes by the periodic law, Fig. 4a, and so their complexities do, Fig. 4b. This contrasts the case of the permittivity monotonic rise when the signal shape becomes intricate quickly, Fig. 5a, that leads to the saturation effect in the behaviour of the complexity, Fig. 5b.



a) b)

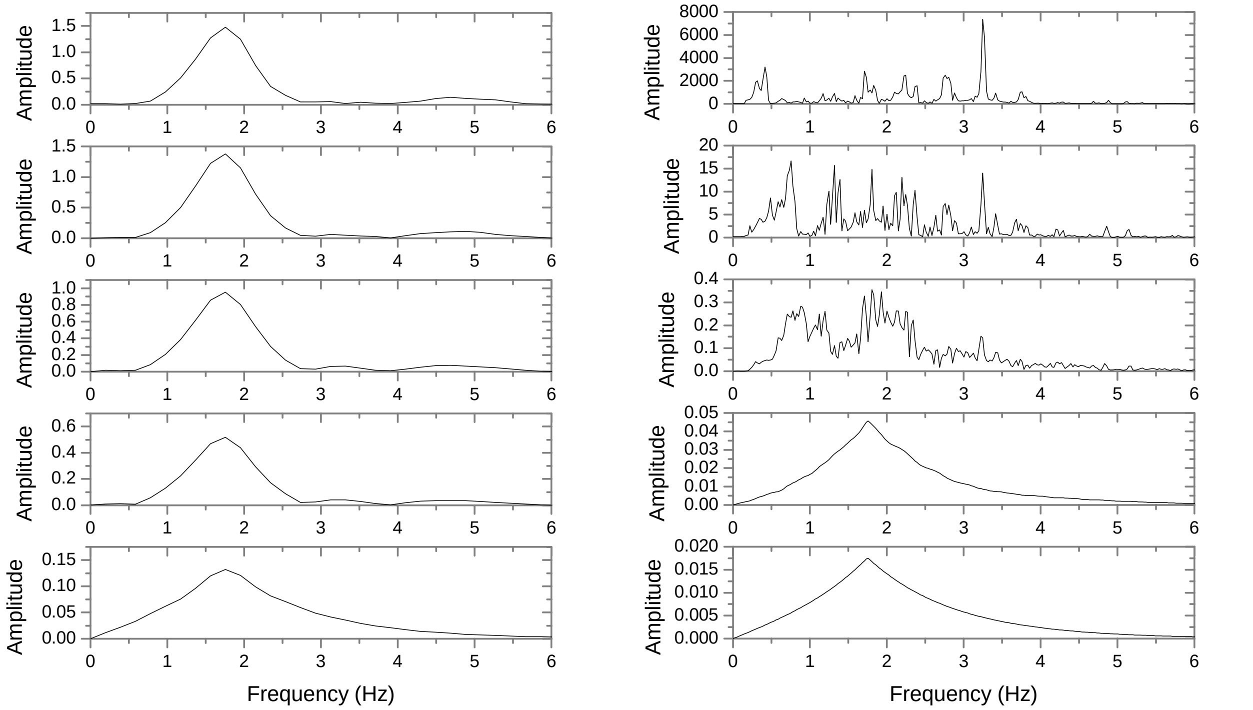
Figure 4. The transformation of the Lorentz-shape signal (a) and its complexity (b) for the periodic medium change.



a) b)

Figure 5. The transformation of the Lorentz-shape signal (a) and its complexity (b) for the monotonic increase of the medium permittivity.

Fig. 6 shows that the periodic medium change has very little influence on the spectrum, only uniform amplification of the whole spectrum by the factor of ten occurs during 20 cycles of the medium changes. Contrary to that a significant distortion of the spectrum is present in the case of the monotonic medium change, Fig. 6b. Spectrum rebuilding begins at about the tenth cycle of the medium change when the permittivity approximately doubles. Then the amplification of some spectrum lines moves to the region of high frequencies and the sharpening of the spectrum becomes apparent and even more pronounced than for the Gaussian-signals. The amplification of some spectrum lines is significantly greater (up to approximately 10^6 times) after the three-fold increase of the permittivity value.



a) b)

Figure 6. *The transformation of the Lorentz-shape signal spectrum after 5, 10, 15 and 20 cycles (from bottom to top) of the medium change for the periodic (a) and monotonic rise (b) laws.*

4. CONCLUSIONS

Transformation of the signals with various initial shapes of different initial complexities is considered for two kinds of the medium permittivity change: a monotonic rise of the relative permittivity and a periodic change. The time medium variation has the form of a sequence of cycles at which the permittivity changes according to various step-wise laws.

It is shown that these variations of the medium lead to a distortion of the signal shape, its spectrum, and its complexity, the measure of the signal informational content. Three groups of signals with different initial shapes and complexities are considered.

Calculation of the signals, their spectra and complexity are performed exactly. It is shown that the growth of the signal complexity is accompanied by sharpening of the signal spectrum and it reveals more strongly in the case of the permittivity monotonic rise when the spectrum becomes distinctly striped than in the case of the periodic medium changes.

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