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INTERACTION OF TIME-VARYING AIRY PULSES WITH A LAYER

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The unique properties of the Airy beams such as non-diffractive, accelerating, and self-healing together with their very interesting applications (transfer of small particles along a parabolic trajectory, light bullet and other) motivates the of investigations Airy pulses in time domain because mainly harmonic time dependence is considered in literature.

In this work pulses with an envelope in the form of the Airy function are obtained using Green's functions for a paraxial equation in 1D and 3D cases in time domain. The interaction of such pulses with a dielectric layer is investigated.

A paraxial equation with a source

$$E = F(t,x,y,z)e^{-ikx} \qquad \left|F''_{xx}\right| << \left|2ik_x F'_x\right|$$

$$\frac{\partial^2 F}{\partial t^2} + 2ikv^2\frac{\partial F}{\partial x} + k^2v^2F - v^2\frac{\partial^2 F}{\partial y^2} - v^2\frac{\partial^2 F}{\partial z^2} = -v^2\mu_0\frac{\partial j}{\partial t}e^{ikx}$$

The Green functions

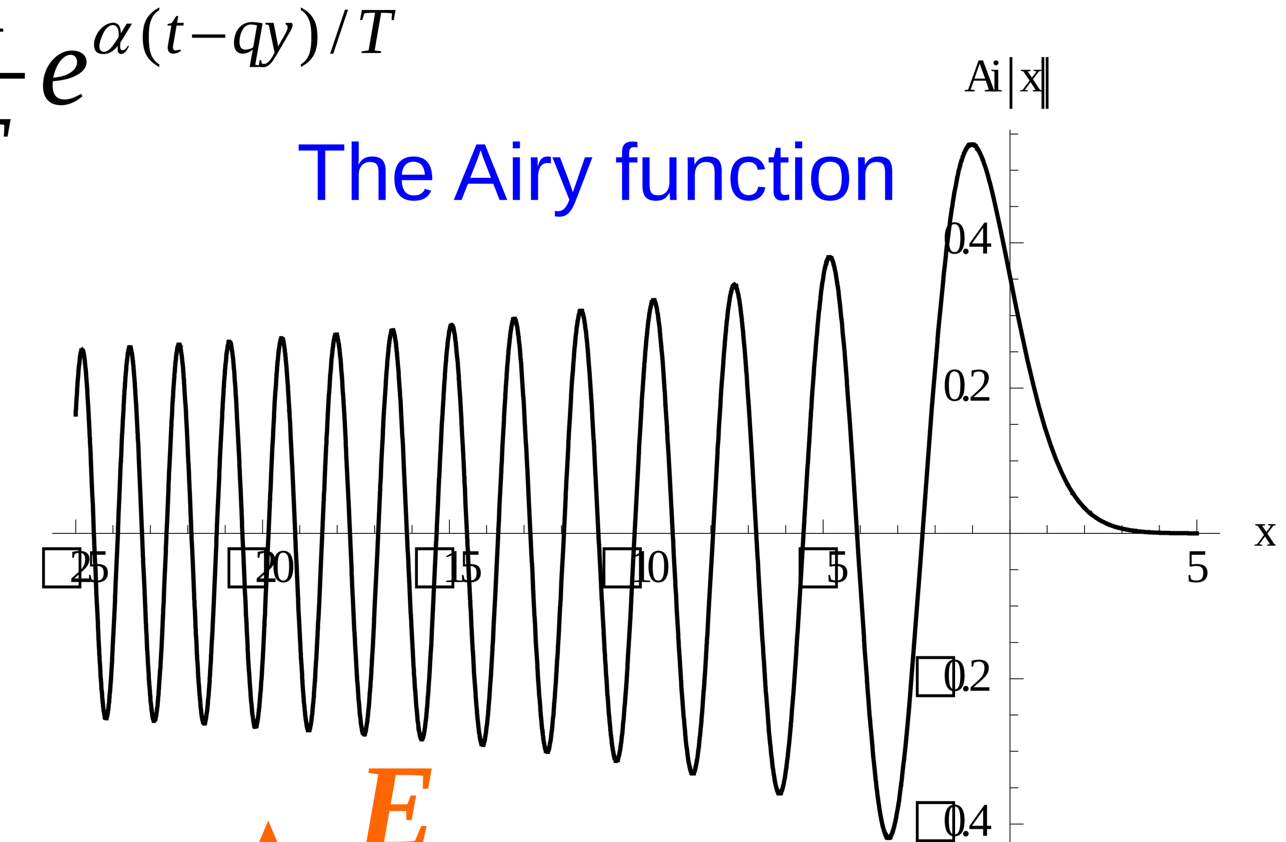
3D: $$G = \frac{-(1-i)v\sqrt{k}}{8\pi x\sqrt{\pi x}}\theta(x)e^{i\frac{k}{2}x + i\frac{kv^2}{2x}\left(t+i\frac{\mu_0\mu\sigma}{2k}x\right)^2 - i\frac{k^2y^2}{2kx} - i\frac{k^2z^2}{2kx}}$$

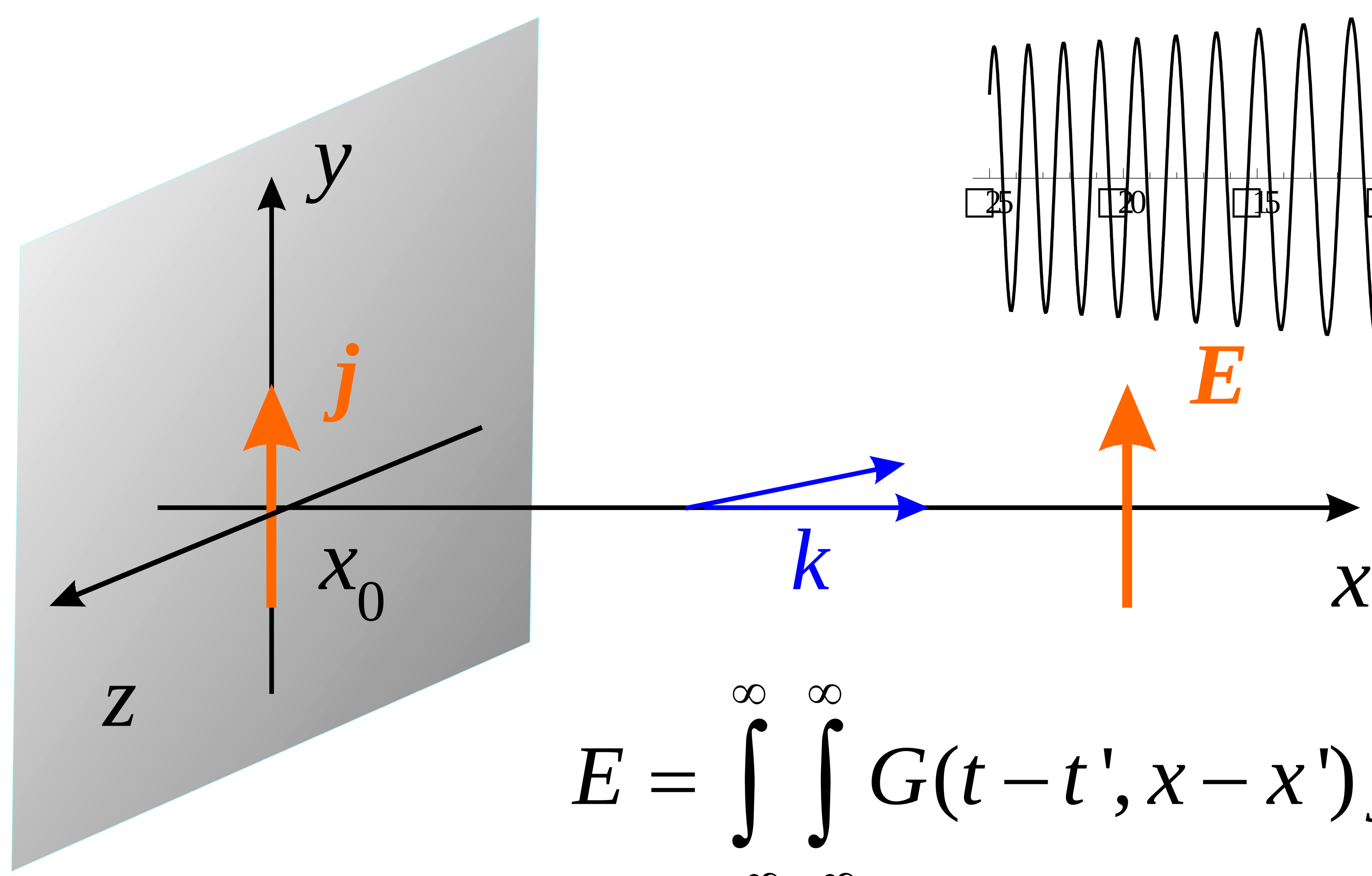
1D: $$G = -\frac{(1-i)v}{4\sqrt{\pi kx}}\theta(x)e^{i\frac{k}{2}x + i\frac{kv^2}{2x}\left(t+i\frac{\mu_0\mu\sigma}{2k}x\right)^2}$$

The current as a source

$$j = j_0\delta(x-x_0)\mathrm{Ai}\left(\frac{t-qy}{T}\right)\frac{1}{T}e^{\alpha(t-qy)/T}$$

The Airy function





$$E = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(t-t', x-x')\, j(t', x')\,dt'\,dx'$$

Generation of decelerated Airy pulses

$$E(t,x)=\theta(x-x_0)\,J\,\frac{\partial}{\partial t}\left[e^{i\Phi(t,x)}\,\mathrm{Ai}\left(\left(t+i\mu_0\mu\sigma v^2T^2X-qy\right)/T-X^2+2i\alpha X\right)\right]$$

$$X=(1-q^2v^2)\,\frac{x-x_0}{2kv^2T^2}$$

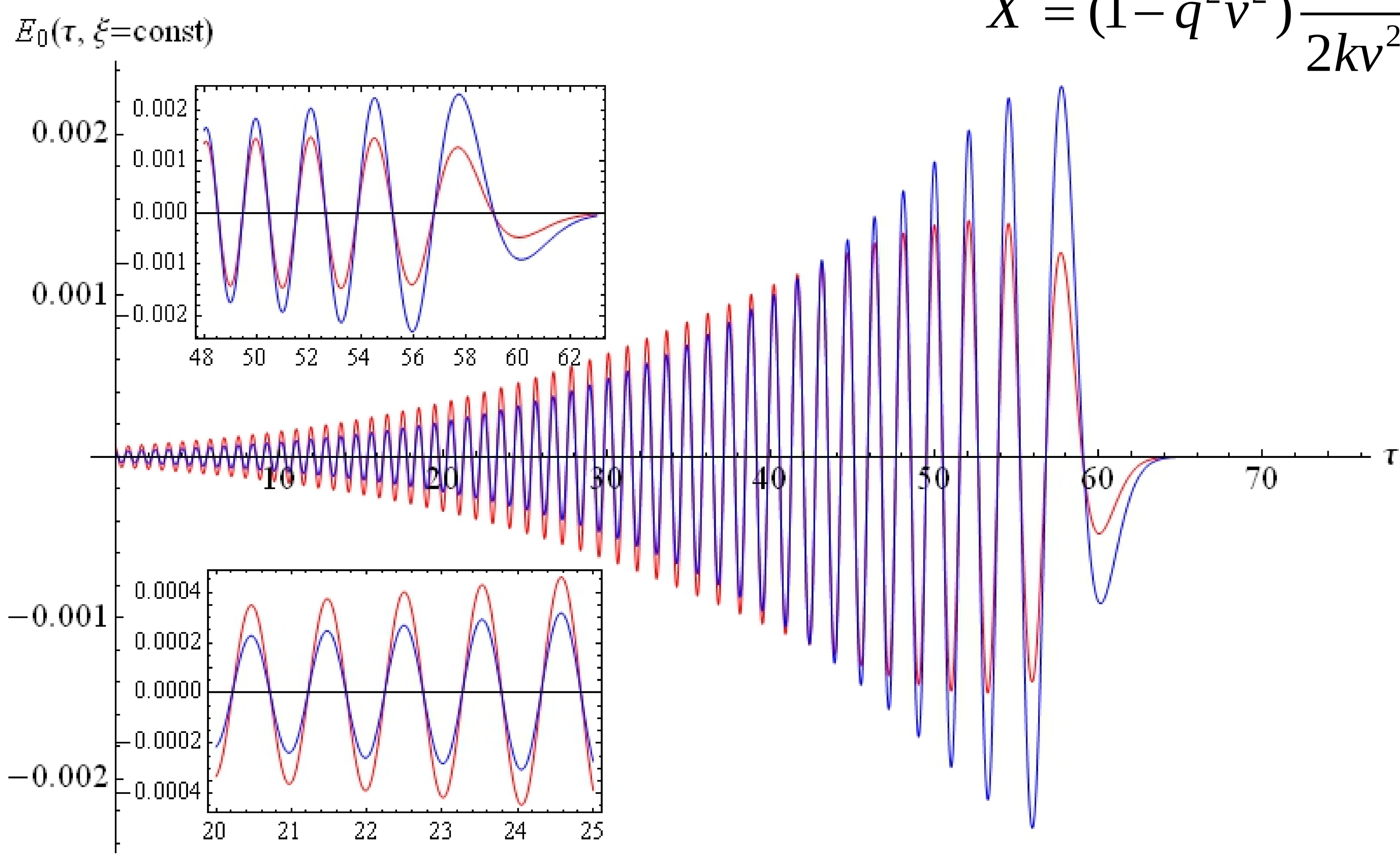
The pulse velocity:

$$\dot{x}=\frac{2k^2v^4T^3}{x-x_0-i\alpha 2kv^2T^2}=$$

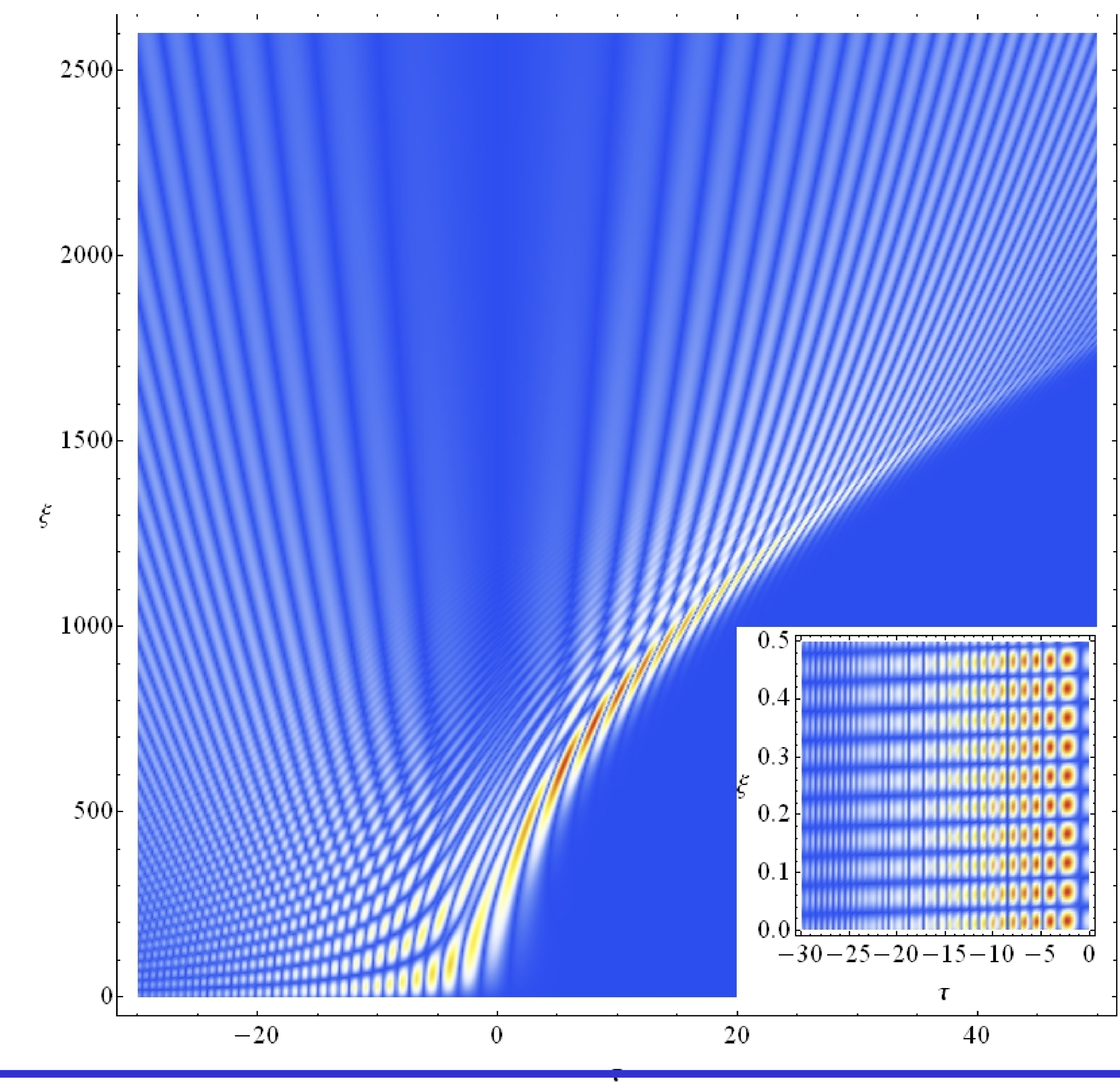
$$=\frac{kv^2T}{\sqrt{t/T-const-\alpha^2}}$$

The pulse deceleration:

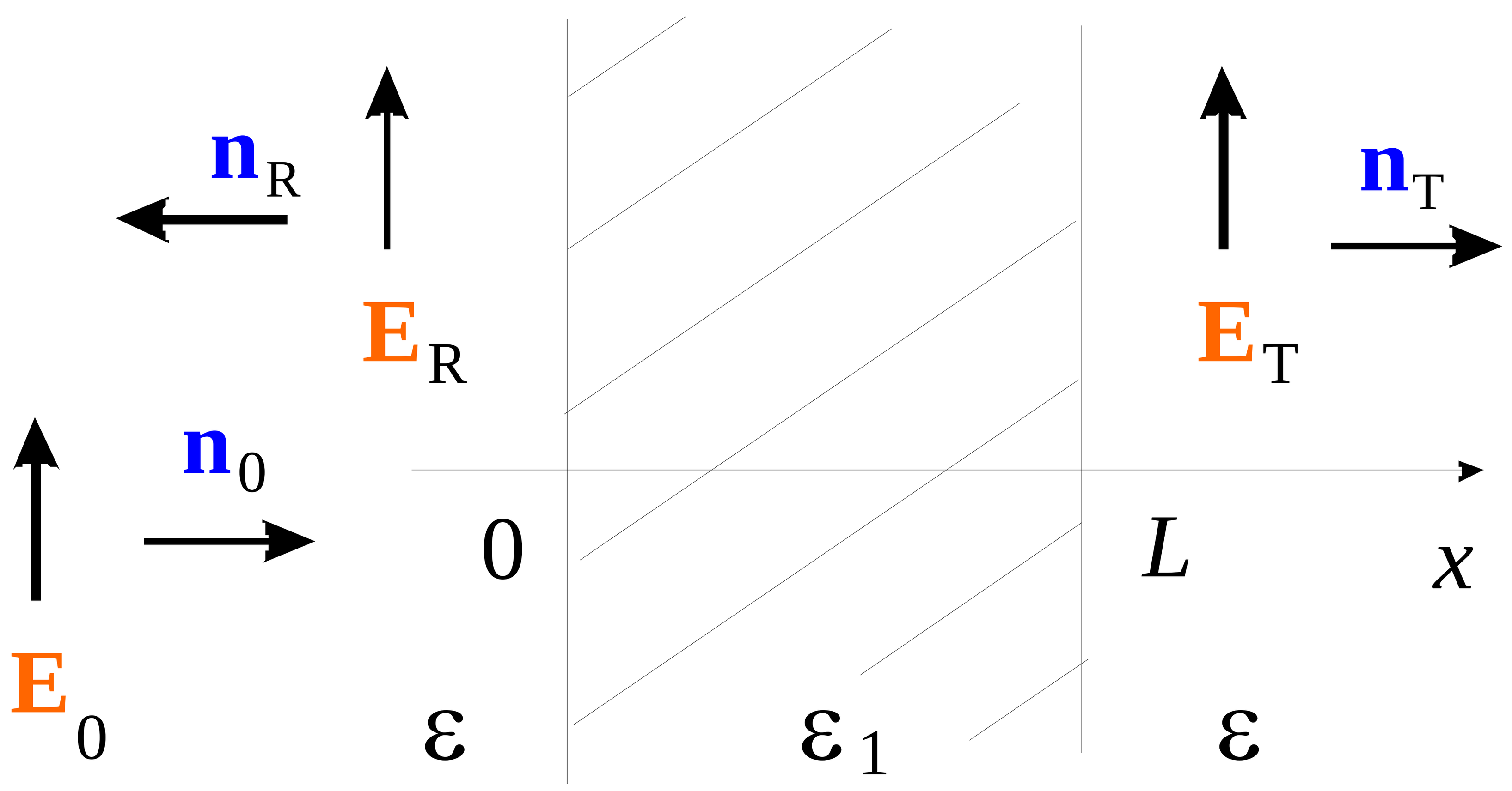
$$\ddot{x}=-\dot{x}^2/(x-x_0)<0$$



The time-spatial trajectories



Airy pulse interaction with a dielectric layer



$$R=\frac{r(1-e^{-i2\eta L\sqrt{\varepsilon_1/\varepsilon}})}{r^2e^{-i2\eta L\sqrt{\varepsilon_1/\varepsilon}}-1};\quad r=\frac{1-\sqrt{\varepsilon_1/\varepsilon}}{1+\sqrt{\varepsilon_1/\varepsilon}};\quad T=\frac{(r^2-1)e^{-ikL(\sqrt{\varepsilon_1/\varepsilon}-1)}}{r^2e^{-i2kL\sqrt{\varepsilon_1/\varepsilon}}-1};$$

A spectral representation of pulses

$$E_0(t,x)=-\frac{\mu_0v^2}{4\pi^2}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}\frac{e^{i\omega t+i(\omega T+i\alpha)^3/3+i\eta(x-x_0)}}{\omega^2+k^2v^2+2\eta kv^2-i\omega\mu_0\sigma v^2}d\eta$$

$$E_R(t,x)=-\frac{\mu_0v^2}{4\pi^2}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}\frac{R(\eta)e^{i\omega t+i(\omega T+i\alpha)^3/3-i\eta(x+x_0)}}{\omega^2+k^2v^2+2\eta kv^2-i\omega\mu_0\sigma v^2}d\eta$$

$$E_T(t,x)=-\frac{\mu_0v^2}{4\pi^2}\frac{\partial}{\partial t}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}\frac{T(\mu)e^{i\omega t+i(\omega T+i\alpha)^3/3+i\eta(x-x_0)}}{\omega^2+k^2v^2+2\eta kv^2-i\omega\mu_0\sigma v^2}d\eta$$

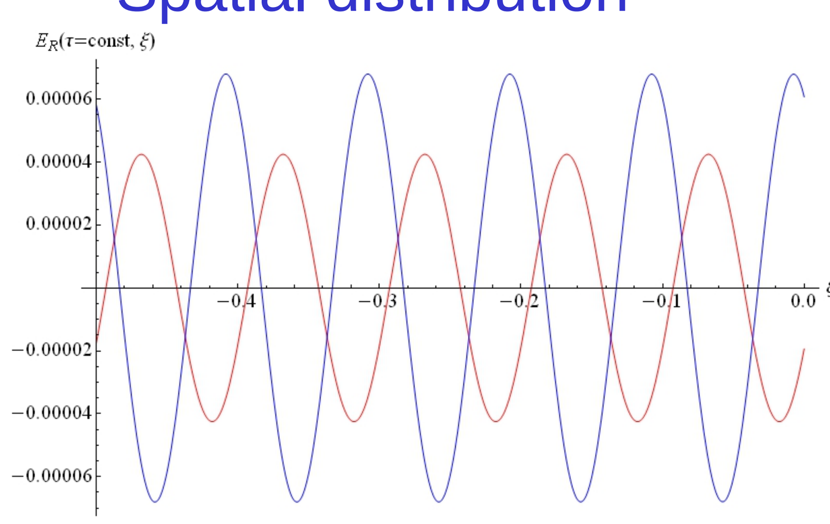
The deceleration of the reflected Airy pulse

$$E_R(t,x)=\mu_0 e^{-i\frac{k}{2}(x-x_0)}\theta(x-x_0)\,r\left(I_0-\sum_{N=0}^{N=\infty}\left[r^{2N}/N!-r^{2(N+1)}/(N+1)!\right]I_N\right)$$

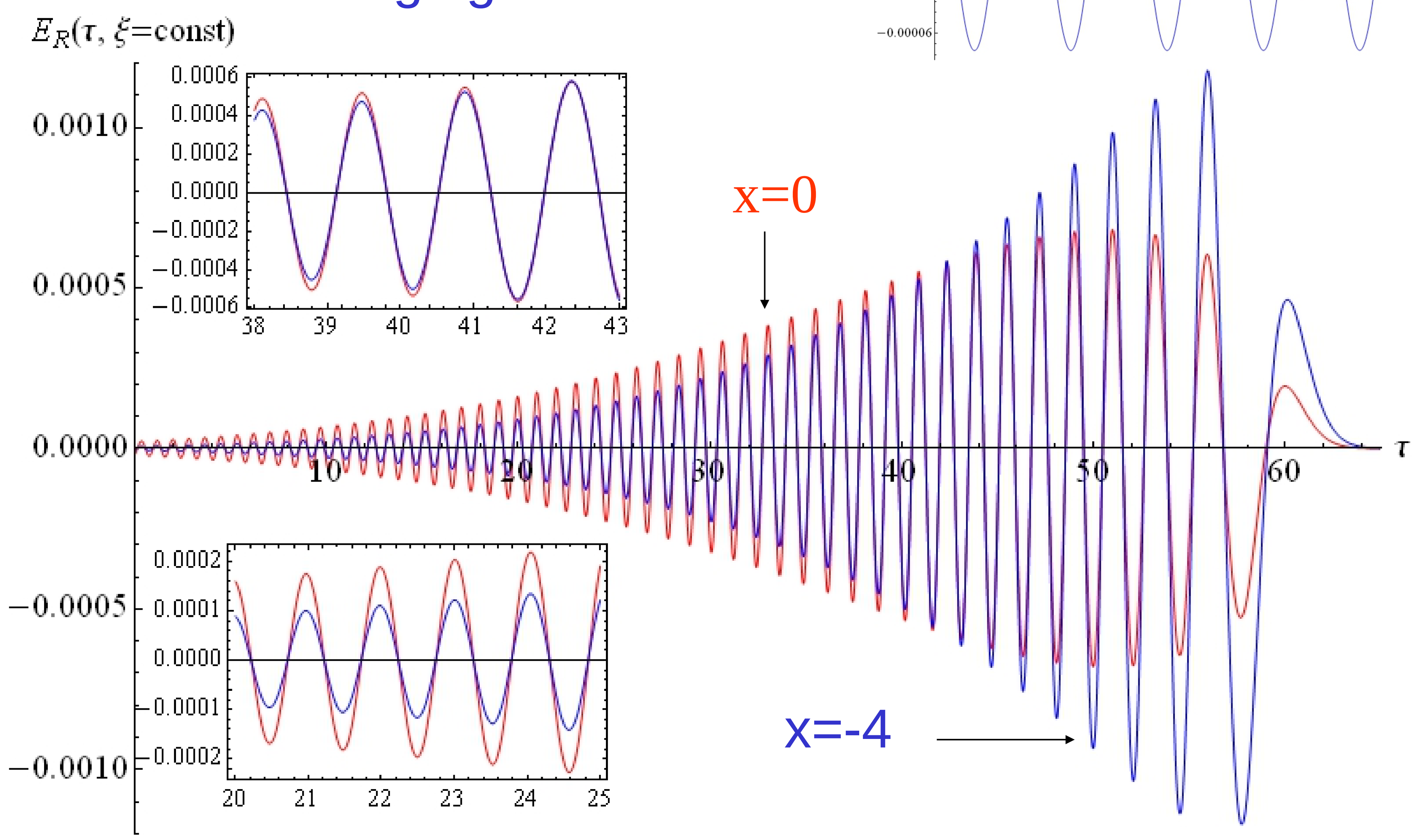
$$I_N=\mathrm{Ai}\left(t/T+i2\alpha X_N-X_N^{\,2}\right)\exp\left((\alpha+iX_N)t/T-i2X_N^{\,3}/3-8\alpha X_N^{\,2}+i\alpha^2X_N\right);$$

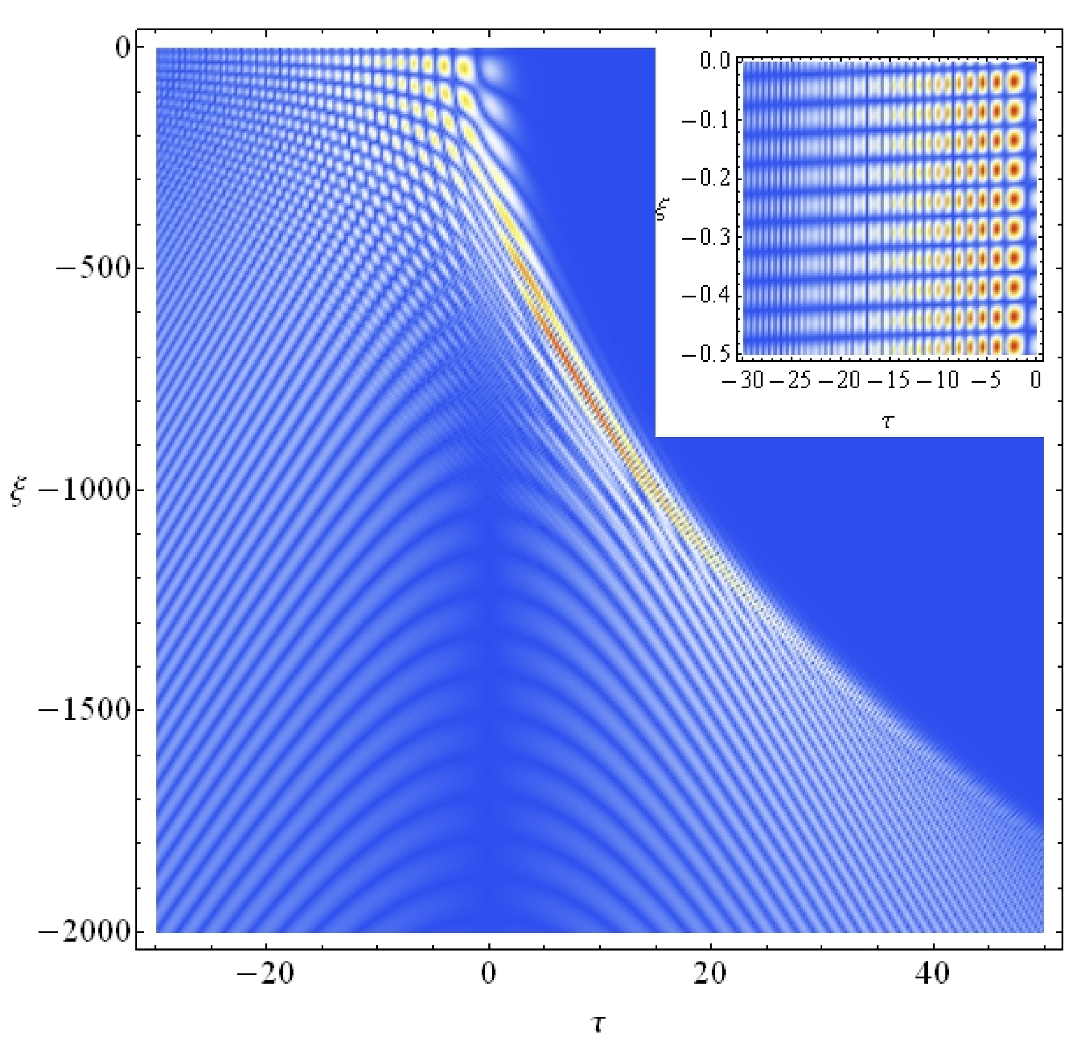
$$X_N=\frac{2NL\sqrt{\varepsilon_1/\varepsilon}+x-x_0}{2kv^2T^2}$$

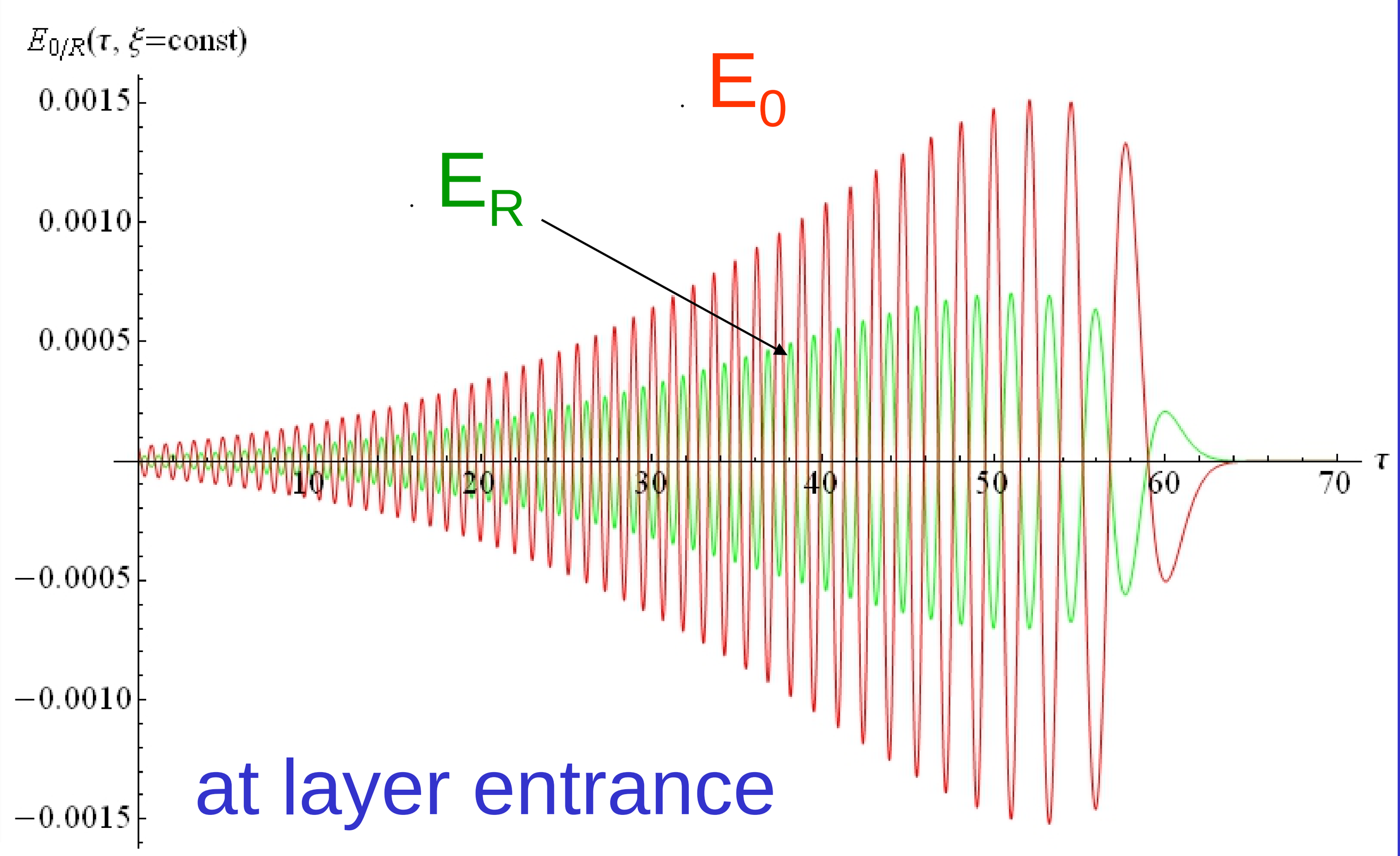
Spatial distribution



Changing in time







at layer entrance

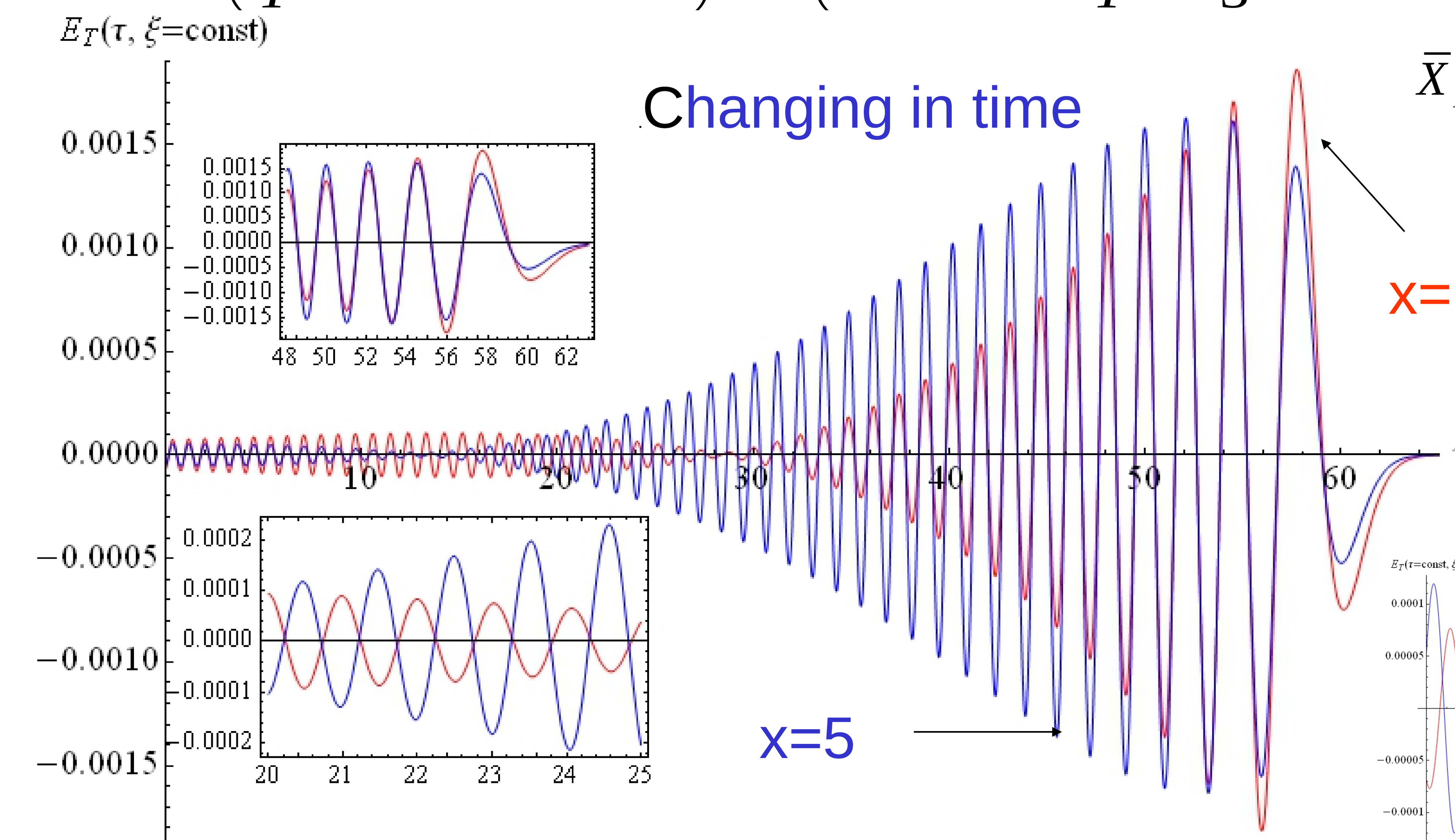
The deceleration of the transmitted Airy pulse

$$E_T(t,x)=\mu_0(r^2-1)e^{-i\frac{k}{2}[(a-1)L+x-x_0]}\sum_{N=0}^{\infty}\frac{1}{N!}r^{2N}e^{-iNkaL}K_{2N+1}$$

$$K_N=\mathrm{Ai}\left(\frac{t}{T}+i2\alpha\bar{X}_N-\bar{X}_N^{\,2}\right)\exp\left(\left[\alpha+i\bar{X}_N\right]\frac{t}{T}-i\frac{2}{3}\bar{X}_N^{\,3}-2\alpha\bar{X}_N^{\,2}+i\alpha^2\bar{X}_N\right);$$

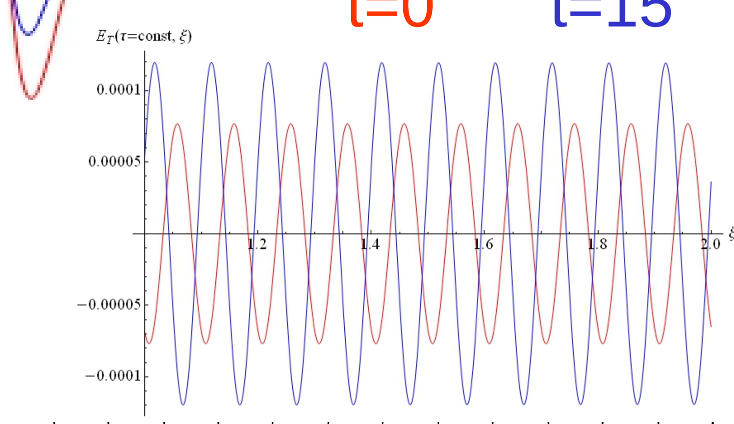
$$\bar{X}_N=\frac{(Na-1)L+x-x_0}{2T^2kv^2}$$

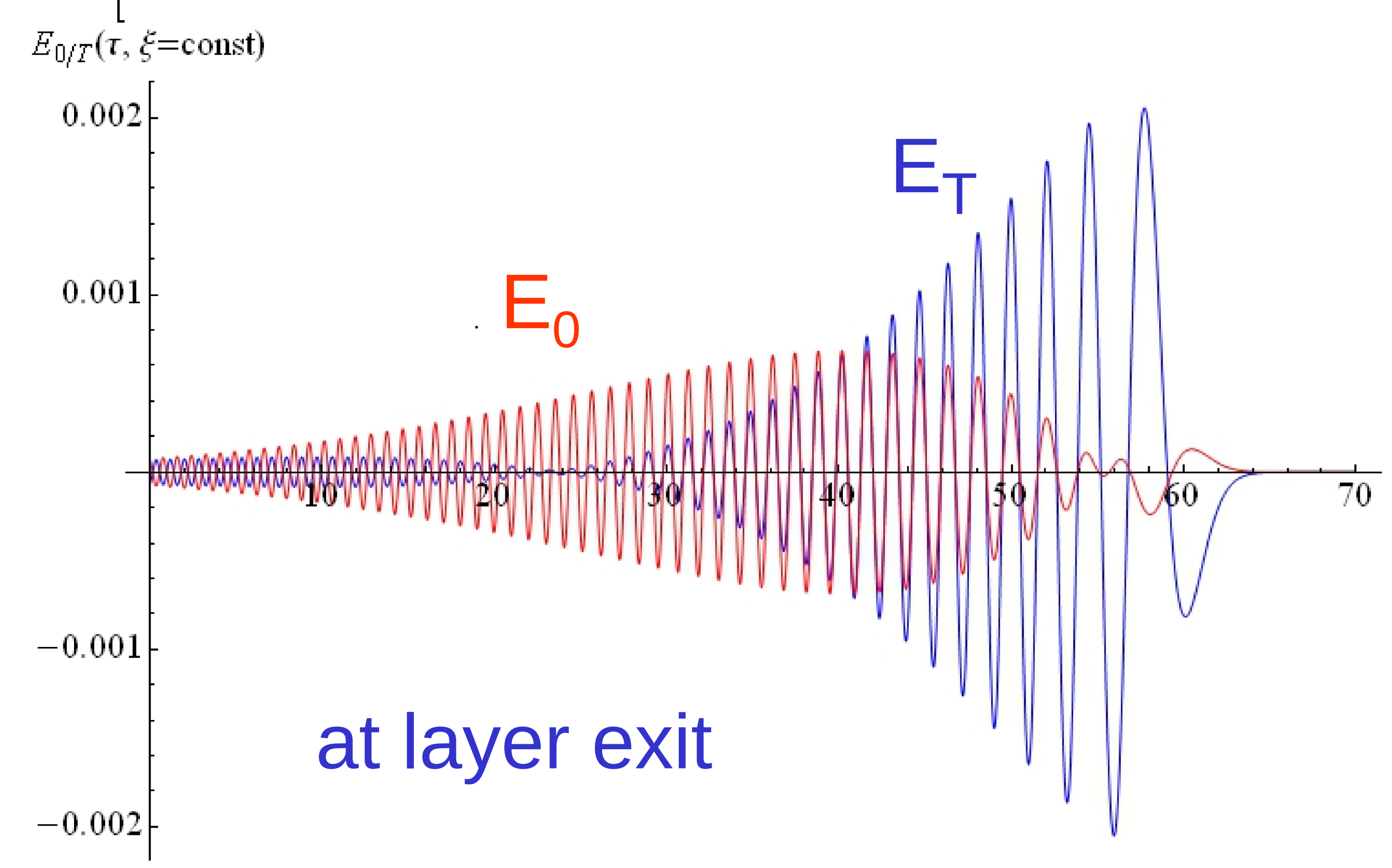
Changing in time



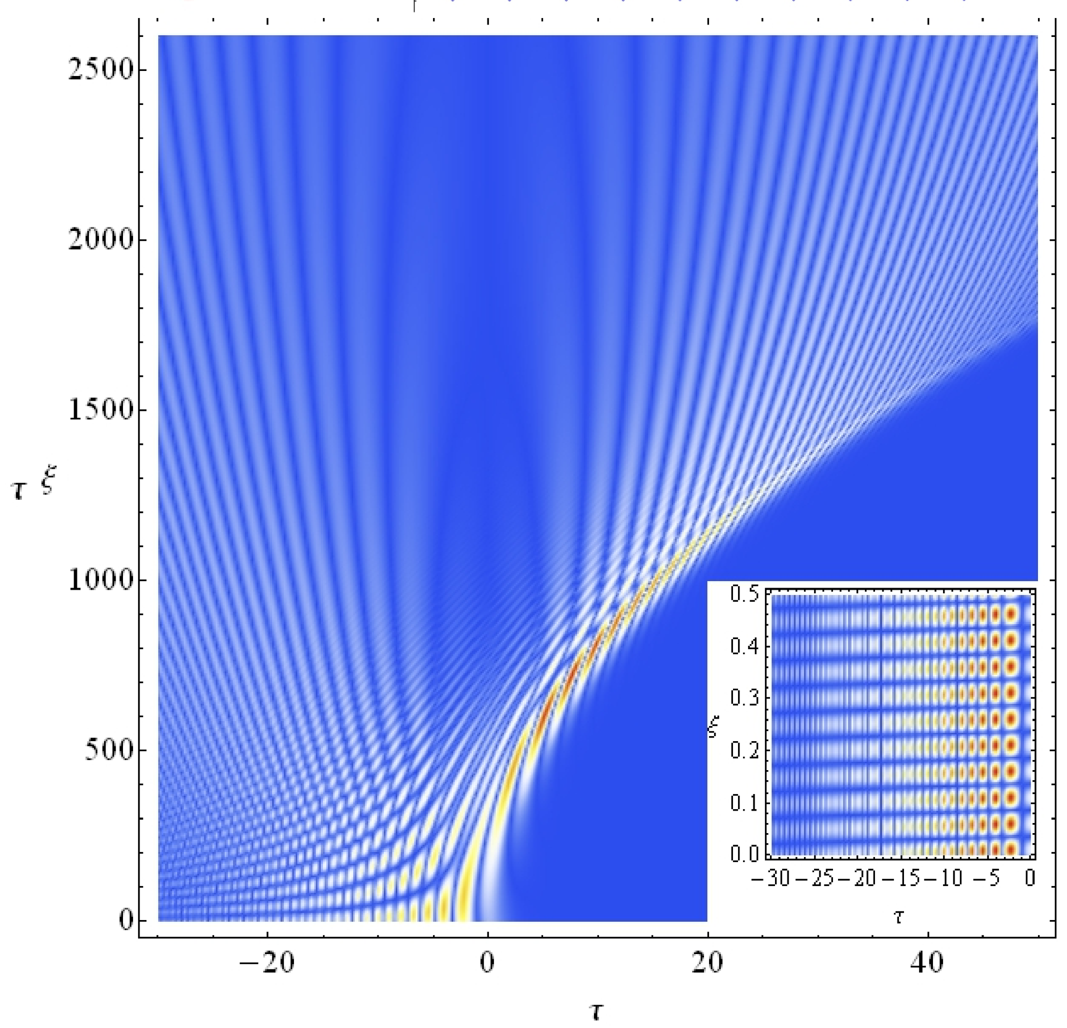
Spatial distribution

t=0 t=15





at layer exit



Conclusions

Time dependent electromagnetic Airy pulses are obtained as the radiation of a special current. The pulses propagate with deceleration and stop at the infinite distance from the source. The pulses reflected from and transmitted through the layer are derived as the sums of the decelerated Airy pulses with arguments shifted in time.